

a. $x^2 - 4 > 0$

$$(x+2)(x-2) > 0$$

$$\begin{array}{ccccc} + & - & + \\ \hline -2 & & 2 \end{array}$$

$$(-\infty, -2) \cup (2, \infty)$$

b. v.a. when denominator?

$$x = 2 \quad x = -2$$

c. h.a. $\lim_{x \rightarrow \pm\infty} \frac{x}{\sqrt{x^2 - 4}} = \pm 1 \quad [y=1 \text{ and } y=-1]$

d. $f'(x) = \frac{1}{\sqrt{x^2 - 4}} \cdot 1 - x \cdot \frac{1}{2}(x^2 - 4)^{-\frac{1}{2}}(2x)$

$$x^2 - 4$$

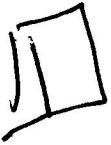
$$= \frac{x^2}{\sqrt{x^2 - 4}}$$

$$x^2 - 4$$

$$\frac{x^2 - 4 - x^2}{\sqrt{x^2 - 4}}$$

$$x^2 - 4$$

$$= \frac{-4}{(x^2 - 4)^{\frac{3}{2}}}$$



1989 AB4
Solution

(a) $x < -2$ or $x > 2$ $\textcircled{+2}$
or $|x| > 2$

(b) $x = 2, x = -2$ $\textcircled{+2}$

(c) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - 4}} = 1$ \textcircled{X}

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - 4}} = -1$$

$$y = 1, y = -1$$

(d)

$$\begin{aligned}f'(x) &= \frac{\sqrt{x^2 - 4} - x \left[\frac{1}{2} (x^2 - 4)^{-1/2} 2x \right]}{x^2 - 4} \\&= \frac{\sqrt{x^2 - 4} - \frac{x^2}{\sqrt{x^2 - 4}}}{x^2 - 4} \\&= \frac{-4}{(x^2 - 4)^{3/2}}\end{aligned}$$

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a. domain range

$$1+6x \geq 0 \quad \sqrt{\text{function}} \Rightarrow y \geq 0$$

$$6x \geq -1 \quad [0, \infty)$$

$$\boxed{x \geq -\frac{1}{6}} \quad [-\frac{1}{6}, \infty)$$

b. $f'(x) = \frac{1}{2}(1+6x)^{-\frac{1}{2}}(6) = \frac{3}{\sqrt{1+6x}}$

$$f'(4) = \frac{3}{\sqrt{1+6 \cdot 4}} = \boxed{\frac{3}{5}} \quad \sqrt{1+6x}$$

c. point $(4, 5)$

$$y - 5 = \frac{3}{5}(x - 4)$$

$$y = \frac{3}{5}x - \frac{12}{5} + 5 = \frac{3}{5}x + \frac{13}{5}$$

$$\boxed{y = \text{int } (0, \frac{13}{5})}$$

d. $\parallel \Rightarrow m = 1$

$$\frac{3}{5} = 1$$

$$\sqrt{1+6x}$$

$$\sqrt{1+6x} = \frac{3}{5}$$

$$(1+6x)^{\frac{1}{2}} = \frac{3}{5}$$

$$x = \frac{3}{5}$$

$$y = \sqrt{1+6 \cdot \frac{3}{5}} = \sqrt{\frac{37}{5}} = 3$$

$$\boxed{(\frac{4}{5}, 3)}$$

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1976 AB1

Solution

(a) The domain of f is $x \geq -\frac{1}{6}$.

The range of f is $y \geq 0$.

(b) $f'(x) = \frac{3}{\sqrt{1+6x}}$

The slope of the tangent line at $x = 4$ is $f'(4) = \frac{3}{5}$.

(c) $f(4) = 5$

The tangent line is $y - 5 = \frac{3}{5}(x - 4)$

Therefore the y -intercept is at $y = \frac{13}{5}$.

(d) The tangent line parallel to $y = x + 12$ has slope 1.

$$f'(x) = \frac{3}{\sqrt{1+6x}} = 1$$

$$9 = 1 + 6x$$

$$x = \frac{4}{3}$$

$$y = \sqrt{1+6\left(\frac{4}{3}\right)} = 3$$

The coordinates of the point are $\left(\frac{4}{3}, 3\right)$.

#7

P when $x=1$, $x^2=1$ So $ax+b=1$ when $x=1$

$$\begin{array}{l} a(1)+b=1 \\ \hline a=1-b \end{array}$$

defn. of cont.

$$f(1) = \lim_{x \rightarrow 1} f(x)$$

$$f(1)=1^2=1 \quad \lim_{x \rightarrow 1^-} f(x)=1^2=1$$

$$\lim_{x \rightarrow 1^+} f(x)=a(1)+b=1-b+0=1$$

so $\lim_{x \rightarrow 1^+} f(x)=1$

same

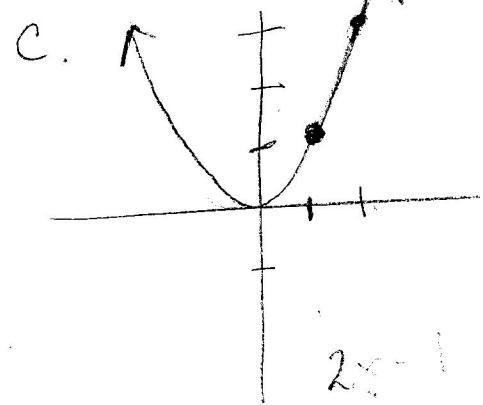
b. $2x=a$ when $x=1$

$$2(1)=a$$

$$2=a$$

$$\text{if } a=2, b=1-a=-1$$

$$f(x)=\begin{cases} x^2, & x \leq 1 \\ 2x-1, & x > 1 \end{cases}$$



$$\lim_{x \rightarrow 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x \rightarrow 1^-} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1^-} (x+1) = 2$$

$$\lim_{x \rightarrow 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x \rightarrow 1^+} \frac{2x-1-1}{x-1} = \lim_{x \rightarrow 1^+} \frac{2(x-1)}{x-1} = 2$$

$$\therefore f(x) = \begin{cases} x^2, & x \leq 1 \\ ax+b, & x > 1 \end{cases} \bullet(1,1) \circ(1, a+b)$$

(a) $a+b=1$
 $\therefore \underline{a=1-b}$

The function is continuous @ 1 when $a=-b$
 because: I - $f(1)=1 \Leftrightarrow$ therefore exists

$$\text{II} - \lim_{x \rightarrow 1^-} f(x) = 1 = \lim_{x \rightarrow 1^+} f(x) \therefore \lim_{x \rightarrow 1} f(x) \text{ exists}$$

$$\text{III} - \lim_{x \rightarrow 1} f(x) = f(1) = 1$$

Therefore the function is continuous @ $x=1$
 & all other values of x , in its domain.

(b) $f'(x) = \begin{cases} 2x, & x \leq 1 \\ a, & x > 1 \end{cases} \bullet(1,2) \quad a=2 \therefore b=1-2=-1$

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ 2x-1, & x < 1 \end{cases}$$

$$\bullet \lim_{h \rightarrow 0^-} \frac{(1+h)^2 - 1^2}{h} = \lim_{h \rightarrow 0^-} \frac{1+2h+h^2-1}{h} = \lim_{h \rightarrow 0^-} 2+h = 2$$

$$\bullet \lim_{h \rightarrow 0^+} \frac{(1+h)^2 - 1^2}{h} = \lim_{h \rightarrow 0^+} \frac{1+2h+h^2-1}{h} = \lim_{h \rightarrow 0^+} 2+h = 2$$

Therefore the function is continuous @ $x=1$
~~SEE PART A~~ \therefore the one sided derivatives
 exist @ $x=1$ since they are equal
 The function is differentiable @ $x=1$.

