

FRQ Practice Unit 5

1993 AB 6

a) $\frac{dp}{dt} = k(800 - p)$

$$\int \frac{dp}{800-p} = \int k dt$$

$$-\ln |800-p| = kt + C$$

$$-\ln |800-500| = k(0) + C$$

$$-\ln 300 = C$$

$$-\ln |800-p| = kt - \ln 300$$

$$\ln |800-p| = -kt + \ln 300$$

e

e

$$800-p = e^{-kt} \cdot e^{\ln 300}$$

$$-p = 300e^{-kt} - 800$$

$$p = -300e^{-kt} + 800$$

b) $700 = -300e^{-k(2)} + 800$

$$-100 = -300e^{-2k}$$

$$\frac{1}{3} = e^{-2k}$$

$$\ln\left(\frac{1}{3}\right) = -2k$$

$$k = \frac{\ln\left(\frac{1}{3}\right)}{-2} \approx .549$$

c) $\lim_{t \rightarrow \infty} P(t) = \boxed{800}$

2012 AB/BC 5

a) when $B = 40$, $\frac{dB}{dt} = \frac{1}{5}(100-40) = 12$

when $B = 70$, $\frac{dB}{dt} = \frac{1}{5}(100-70) = 6$

Since $B = 40$ gives a higher rate of change, the bird is gaining weight faster when it weighs

$\boxed{40 \text{ grams.}}$

b) $\frac{d^2B}{dt^2} = \frac{1}{5}(-1)\left(\frac{dB}{dt}\right) = -\frac{1}{5}\left(\frac{1}{5}\right)(100-B) = \boxed{-\frac{1}{25}(100-B)}$

For $20 < B < 100$, $\frac{d^2B}{dt^2}$ is negative. So the

graph would be concave down. The given graph has a portion that is concave up.

$$c) \frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\int \frac{dB}{100 - B} = \int \frac{1}{5} dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

$$-\ln|100 - 20| = \frac{1}{5}(0) + C$$

$$-\ln 80 = C$$

$$-\ln|100 - B| = \frac{1}{5}t - \ln 80$$

$$\ln|100 - B| = -\frac{1}{5}t + \ln 80$$

$$e^{100 - B} = e^{-\frac{1}{5}t} \cdot e^{\ln 80}$$

$$-B = 80e^{-\frac{1}{5}t} - 100$$

$$\boxed{B = -80e^{-\frac{1}{5}t} + 100}$$

2012 BC4

a) point (1, 15) slope $f'(1) = 8$

$$\boxed{y - 15 = 8(x - 1)}$$

$$y = 15 + 8(1.4 - 1) = \boxed{18.2}$$

b) subinterval length = $\frac{1.4 - 1}{2} = 0.2$

$$.2(10) + .2(13) = \boxed{4.6}$$

$$\int_1^{1.4} f'(x) dx = f(1.4) - f(1)$$

$$f(1.4) = \int_1^{1.4} f'(x) dx + f(1) = 4.6 + 15 = \boxed{19.6}$$

x	y	dy/dx
1	15	8
1.2	16.6	12
1.4	19	

$$15 + 8(.2) = 16.6$$

$$16.6 + 12(.2) = 19$$

$$\boxed{f(1) \approx 19}$$

2011 BC5

a) point (0, 1400)

$$\text{slope } \frac{dW}{dt} = \frac{1}{25} (1400 - 300) = 44$$

$$\text{tangent line: } \boxed{y - 1400 = 44(x - 0)}$$

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = \boxed{1411 \text{ tons}}$$

$$b) \frac{d^2W}{dt^2} = \frac{1}{25} (1) \cdot \frac{dW}{dt} = \frac{1}{25} \cdot \frac{1}{25} (W - 300) = \boxed{\frac{1}{625} (W - 300)}$$

For $0 \leq t \leq \frac{1}{4}$, $\frac{d^2W}{dt^2}$ is positive b/c $W \geq 1400$.

Since $w(t)$ is concave up, the answer in part a is an underestimate.

$$c) \frac{dW}{dt} = \frac{1}{25} (W - 300)$$

$$\int \frac{dW}{W - 300} = \int \frac{1}{25} dt$$

$$\ln |W - 300| = \frac{1}{25} t + C$$

$$\ln |1400 - 300| = \frac{1}{25}(0) + C$$

$$\ln 1100 = C$$

$$\ln |W - 300| = \frac{1}{25} t + \ln 1100$$

$$e^{\ln |W - 300|} = e^{\frac{1}{25} t + \ln 1100}$$

$$\boxed{W = 300 + 1100e^{\frac{1}{25}t}}$$

2000 BC 6

a) anywhere $y=1 \Rightarrow dy/dx = 0$
 anywhere $x=0 \Rightarrow dy/dx = 0$

x	y	dy/dx
1	0	1
-1	0	-1
1	-1	4
-1	-1	-4

b) when $y=1$ in part a, $dy/dx = 0$. The graph in part b does not have a slope of 0 when $y=1$

c) $\frac{dy}{dx} = x(y-1)^2$

$$\int \frac{dy}{(y-1)^2} = \int x dx$$

$$u = y-1$$

$$\frac{du}{dy} = 1 \quad du = dy$$

$$\int \frac{du}{u^2} = \int u^{-2} du = -u^{-1} + C$$

$$\frac{-1}{y-1} = \frac{1}{2}x^2 + C$$

$$\frac{-1}{-2} = \frac{1}{2}(0)^2 + C$$

$$\frac{1}{2} = C$$

$$\frac{-1}{y-1} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\frac{1}{y-1} = -\frac{1}{2}x^2 - \frac{1}{2}$$

$$y-1 = \frac{1}{-\frac{1}{2}x^2 - \frac{1}{2}} = \frac{2}{-x^2-1}$$

$$y = 1 + \frac{2}{-x^2-1} \quad \text{or} \quad 1 - \frac{2}{x^2+1}$$

d) range is $[-1, 1)$

b/c $f(0) = -1$ b/c $y = 1 - \frac{2}{x^2+1}$ has a horiz. asymptote at $y=1$