

## Practice -- Group Ranking Methods

1. 21 members of a fraternity are planning on watching sports on a Sunday afternoon. Unfortunately, there are 5 big games being played at the same time, and the house only has one TV. The president of the fraternity decides to have the members of the fraternity vote. The preference ballots are shown below. (B = Basketball, H = Hockey, N = Nascar, F = Football, S = Soccer.)

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
N	H	B	H	N	F	H	B	H	F	F
S	F	S	B	S	N	B	S	B	N	N
F	B	H	N	F	B	N	H	N	B	B
H	N	N	F	H	S	F	N	F	H	S
B	S	F	S	B	H	S	F	S	S	H

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
H	F	F	H	N	H	B	N	H	N
F	N	N	B	S	F	S	S	B	S
B	B	B	N	F	B	H	F	N	F
N	H	S	F	H	N	N	H	F	H
S	S	H	S	B	S	F	B	S	B

- A. Give the preference schedule for this election.  
 B. Determine the plurality winner. **Hockey**  
 C. Is the plurality winner a majority winner? **no**

$$\frac{1}{2}(21) = 10.5 \text{ maj} \geq 11$$

5	3	3	5	3	2
N	H	B	H	F	F
S	F	S	B	N	N
F	B	H	N	B	B
H	N	N	F	S	H
B	S	F	S	H	S

N 5  
 H 4  
 B 3  
 F 3  
 S 5

2. Determine the plurality and plurality with elimination winner:

<u>plurality</u>	Number of voters	7	7	8	5
A	First choice	A	C	D	B
B	Second choice	B	B	C	A
C	Third choice	C	A	B	C
D	Fourth choice	D	D	A	D

A 7  
 B 5  
 C 7  
 D 8

**D wins**

<u>plurality w/elim</u>	Round 1	Round 2	Round 3
A	7	12	19
B	5	7	8
C	7	8	8
D	8	8	8

**A wins**

3. Determine the Plurality, Borda, Runoff, and Sequential Runoff winners for the following preference schedule.

↑	↑	↑	↑
A	B	C	C
D	D	B	D
C	A	D	A
B	C	A	B
16	20	12	7

<u>plurality</u>	A	16
B	20	
C	19	
D	0	

**B wins**

Borda

A:  $16(4) + 0(3) + 27(2) + 12(1) = 130$   
 B:  $20(4) + 12(3) + 0(2) + 23(1) = 139$   
 C:  $19(4) + 0(3) + 16(2) + 20(1) = 128$   
 10D:  $0(4) + 43(3) + 12(2) + 0(1) = 153$

**D wins**

Run-off

A	16	B	20
B	20	C	35
C	19	D	0
D	0	C	35

**C wins**

4. Each year the Heisman Trophy recognizes one of the country's most outstanding college football players. Each person voting selects a player to rank first, second, and third.

2008 Heisman Trophy Votes

Player	1 <sup>st</sup> Place	2 <sup>nd</sup> Place	3 <sup>rd</sup> Place	Total Points
Bradford	300	315	196	1,726
McCoy	266	288	230	1,604
Tebow	309	207	234	1,575

A. How many points are awarded for a first-place vote? For a second place? For a third place vote?   
 3 points, 2 pts, 1 pt.

B. Would the ranking produced by this system have differed if the plurality method had been used? Explain.

B - 300  
M - 266  
T - 309

Tebow would have won

5. Determine the pairwise comparison winner.

A | B    A | C    A | D  
39 | 43    39 | 43    79 | 3

B | C    B | D    C | D  
69 | 13    79 | 3    79 | 3

	Number of Votes			
	36	30	13	3
1 <sup>st</sup>	A	B	C	D
2 <sup>nd</sup>	B	C	B	A
3 <sup>rd</sup>	C	A	A	B
4 <sup>th</sup>	D	D	D	C

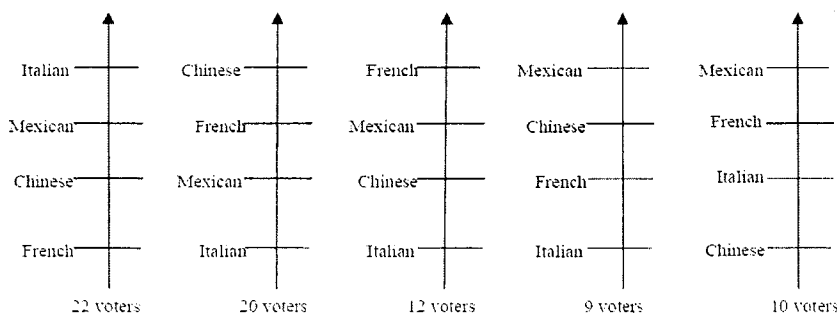
A - 1    C - 2    B wins  
B - 3    D - 0

6. Imagine a ski club with 9 people. Suppose they need to choose the destination for their winter ski trip. Members of the club nominate 5 different different resorts, and they all hold an approval vote. The results are below. Which resort wins?

Resorts	Ballots										
	#1	#2	#3	#4	#5	#6	#7	#8	#9		
Tussey		x			x						2
Montage	x	x			x			x			4
Snowshoe	x		x	x		x		x			5
Lake Tahoe	x	x		x	x	x	x		x		7
Beaver			x				x	x	x		4

Lake Tahoe wins

7. Apply Condorcet's method to the preference schedule.



I | M    I | C    I | E  
22 | 51    32 | 41    22 | 51

M | C    M | E    C | E  
53 | 20    41 | 32    51 | 22

M - 3    C - 2    F - 1    I - 0

M wins

8. Ms. Powell decides to order drinks for your class on the basis of your class vote. In doing so, she selects the preference schedule of a single student (her teacher's pet). Which of Arrow's conditions are violated by this method of determining a group ranking?

nondictatorship

9. Ms. Powell decides to place all of the individual preferences in a hat and draws one. If this method was repeated, would the same group ranking result? Which of Arrow's conditions does this method violate?

nondictatorship & uniqueness of group ranking  
(if repeated may have

10. Do any of Arrow's conditions require that the voting mechanism include a secret ballot? Is a secret ballot desirable in all group-ranking situations? Explain.

different result

you want to know how others <sup>no</sup> vote  
voting procedure may require additional  
actions from each voter (pw voting)