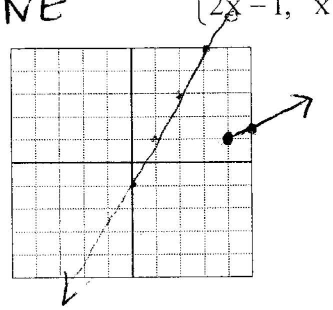
Introduction to Calculus—test review

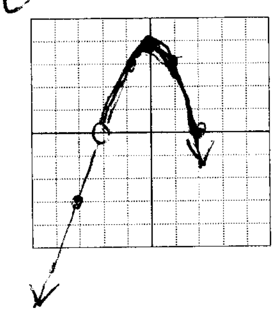
Evaluate the following limits. If the limit does not exist, give the direction (if it has one).

1. $\lim\limits_{x\to 0^-}\frac{1}{x}$ $-\infty$
2. $\lim\limits_{x\to 0^+}\frac{1}{x}$ ∞
3. $\lim\limits_{x\to 0}\frac{1}{x}$ DNE
4. $\lim\limits_{x\to \infty}\frac{1}{x}$ 0
5. $\lim\limits_{x\to 0}\frac{1}{x^2}$ ∞
6. $\lim\limits_{x\to \infty}\frac{2-6x}{5x+1}$ $\frac{-6}{5}$
7. $\lim\limits_{x\to -\infty}\frac{x}{x^3+2}$ 0
8. $\lim\limits_{x\to \infty}\frac{2x^2+1}{x}$ ∞
9. $\lim\limits_{x\to 0}\frac{6x-9}{x^3-12x+3}$ -3
10. $\lim\limits_{x\to 6}\frac{x+6}{x^2-36}$ DNE
11. $\lim\limits_{x\to -2}\frac{x^2-4x+4}{x^2+x-6}$ -4
12. $\lim\limits_{x\to \infty}3$ 3
13. $\lim\limits_{x\to 9}\frac{9-x}{3-\sqrt{x}}$ 6
14. $\lim\limits_{x\to -3}\frac{x^2-9}{x+3}$ -6
15. $\lim\limits_{x\to 4}f(x)$, $f(x)=\begin{cases}\frac{1}{2}x-1, & x\ge 4\\ 2x-1, & x<4\end{cases}$
DNE



jump discont. at $x=4$

16. $\lim\limits_{x\to -2}f(x)$, $f(x)=\begin{cases}-x^2+4, & x>-2\\ 3x+6, & x<-2\end{cases}$ hole at $x=-2$
0



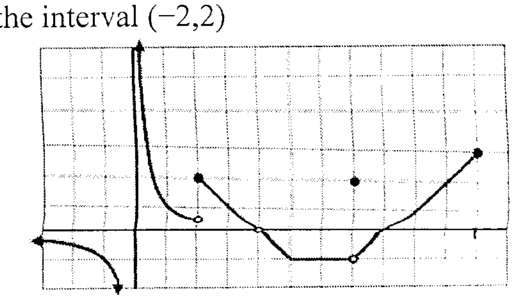
Refer to the graph to evaluate the following:

1. ∞	$\lim\limits_{x\to 0^+}f(x)$	2. $-\infty$	$\lim\limits_{x\to 0^-}f(x)$	3. DNE	$\lim\limits_{x\to 0}f(x)$
4. 2	$\lim\limits_{x\to 2^+}f(x)$	5. 1/2	$\lim\limits_{x\to 2^-}f(x)$	6. DNE	$\lim\limits_{x\to 2}f(x)$
7. -1	$\lim\limits_{x\to 7^+}f(x)$	8. -1	$\lim\limits_{x\to 7^-}f(x)$	9. -1	$\lim\limits_{x\to 7}f(x)$
10. DNE	$\lim\limits_{x\to 11^+}f(x)$	11. 3	$\lim\limits_{x\to 11^-}f(x)$	12. DNE	$\lim\limits_{x\to 11}f(x)$
13. 0	$\lim\limits_{x\to 4}f(x)$	14. -1	$\lim\limits_{x\to 5}f(x)$	15. DNE	$f(0)$
16. 2	$f(2)$	17. DNE	$f(4)$	18. 2	$f(7)$

19. true True or False: $\lim\limits_{x\to c}f(x)$ exists for every c in the interval (2,10)

20. false True or False: $\lim\limits_{x\to c}f(x)$ exists for every c in the interval (−2,2)

21. List each x value where a discontinuity occurs and describe the type of discontinuity.



$x=0$ infinite
$x=2$ jump
$x=4$, $x=7$ hole (removeable)

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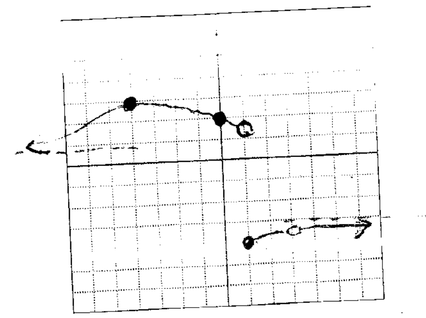
Draw a graph with the following conditions:

- f(0) = 2
- f(1) = -4
- f(-4) = 3
- at f(1) there is a non-removable discontinuity (jump)
- at f(3) there is a removable discontinuity (hole)
- $\lim_{x\to-\infty} f(x) = 1$
- $\lim_{x\to\infty} f(x) = -3$



Use the limit definition to find each derivative:

a. f(x) = 3x² – 5x + 1

$$\lim_{h\to0}\frac{3(x+h)^2-5(x+h)+1-(3x^2-5x+1)}{h} = \lim_{h\to0}\frac{3x^2+6hx+3h^2-5x-5h+1-3x^2+5x-1}{h}$$

$$=\lim_{h\to0}\frac{6hx+3h^2-5h}{h} = \lim_{h\to0} 6x+3h-5 = \boxed{6x-5}$$

b. $g(x) = \frac{-3}{x+2}$

$$g'(x) = \lim_{h\to0}\frac{\frac{-3}{x+h+2}-\frac{-3}{x+2}}{h} = \lim_{h\to0}\frac{\frac{-3(x+2)+3(x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$=\lim_{h\to0}\frac{-3x-6+3x+3h+6}{(x+h+2)(x+2)}\cdot\frac{1}{h} = \lim_{h\to0}\frac{3h}{(x+h+2)(x+2)}\cdot\frac{1}{h}$$

$$=\lim_{h\to0}\frac{3}{(x+h+2)(x+2)} = \boxed{\frac{3}{(x+2)^2}}$$

c. $p(x) = \sqrt{3x+2}$

$$p'(x) = \lim_{h\to0}\frac{\sqrt{3(x+h)+2}-\sqrt{3x+2}}{h} = \lim_{h\to0}\frac{\left(\sqrt{3(x+h)+2}-\sqrt{3x+2}\right)\left(\sqrt{3(x+h)+2}+\sqrt{3x+2}\right)}{h\left(\sqrt{3(x+h)+2}+\sqrt{3x+2}\right)}$$

$$=\lim_{h\to0}\frac{3x+3h+2-(3x+2)}{h\left(\sqrt{3(x+h)+2}+\sqrt{3x+2}\right)} = \lim_{h\to0}\frac{3}{\sqrt{3(x+h)+2}+\sqrt{3x+2}} = \boxed{\frac{3}{2\sqrt{3x+2}}}$$

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Find the derivative of each function:

a. y = 9

$y' = 0$

b. $y = 5x^3 - 4x^2 + 3x - 11$

$y' = 15x^2 - 8x + 3$

c. $f(x) = \frac{1}{x} + \frac{3}{x^2}$

$f'(x) = -\frac{1}{x^2} - \frac{6}{x^3}$

d. $g(x) = (3x^2 + 4)^2 = (9x^4 + 24x^2 + 16)$

$g'(x) = 36x^3 + 48x$

e. $y = \frac{3x^2 - 2x + 7}{5x^3 - 2x}$

f. $f(x) = (4x^2 - 8x + 9)(5x^4 - 6x^3 + 7x^2 - 10x + 21)$

g. $y = 7\sqrt[3]{x^2} + \frac{2}{3}\sqrt{x} = 7x^{2/3} + \frac{2}{3}x^{1/2}$

$y' = \frac{14}{3}x^{-1/3} + \frac{1}{3}x^{-1/2} = \frac{14}{3\sqrt[3]{x}} + \frac{1}{3\sqrt{x}}$

h. $y = \frac{2x^4}{x-9}$

i. $f(x) = 3x^2(7x^3 - 8x + 3) = 21x^5 - 24x^3 + 9x^2$

$f'(x) = 105x^4 - 72x^2 + 18x$

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