KEY

AP Calculus AB

Evaluating Limits Algebraically

Find each limit. You may need to show work to support your answer.

1) $\lim_{x\to 2} x^4$

$2^4 = \boxed{16}$

2) $\lim_{x\to -3}\left(2x^3 - 5x + 7\right)$

$2(-3)^3 - 5(-3) + 7$

$\boxed{-32}$

3) $\lim_{x\to 1}\frac{x-3}{x^2+4}$

$\frac{1-3}{1^2+4} = \boxed{\frac{-2}{5}}$

4) $\lim_{x\to 3}\sqrt{x+1}$

$\sqrt{3+1} = \boxed{2}$

5) $\lim_{x\to 5}\frac{x-5}{x^2-25}$

$\lim_{x\to 5}\frac{x-5}{(x-5)(x+5)}$

$\lim_{x\to 5}\frac{1}{x+5}$

$\boxed{\frac{1}{10}}$

6) $\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}$

$\lim_{x\to -3}\frac{(x+3)(x-2)}{(x+3)(x-3)}$

$\frac{-3-2}{-3-3} = \boxed{\frac{5}{6}}$

7) $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}$

$\lim_{x\to 4}\frac{(\sqrt{x+5}-3)(\sqrt{x+5}+3)}{(x-4)(\sqrt{x+5}+3)}$

$\lim_{x\to 4}\frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \boxed{\frac{1}{6}}$

8) $\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\left(\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}\right)$

$\lim_{x\to 0}\frac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})}$

$\frac{1}{\sqrt{5}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$

9) $\lim_{x\to 0}\frac{\frac{1}{(x+3)}-\frac{1}{3}}{x}$

$\lim_{x\to 0}\frac{\frac{3-(x+3)}{3(x+3)}}{x}$

$\lim_{x\to 0}\frac{\frac{-x}{3(x+3)}}{x} = \lim_{x\to 0}\frac{-1}{3(x+3)} = \boxed{-\frac{1}{9}}$

10) $\lim_{\Delta x\to 0}\frac{2(x+\Delta x)-2x}{\Delta x}$

see below

11) $\lim_{x\to 0}\frac{\sin x}{5x}$

$\lim_{x\to 0}\left(\frac{1}{5}\cdot\frac{\sin x}{x}\right)$

$= \frac{1}{5}\cdot 1 = \boxed{\frac{1}{5}}$

12) $\lim_{x\to 0}\frac{\sin x(1-\cos x)}{2x^2}$

$\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{(1-\cos x)}{x}\cdot\frac{1}{2}\right)$

$= 1\cdot 0\ \frac{1}{2} = \boxed{0}$

13) $\lim_{x\to 0}\frac{\sin^2 x}{x}$

$\frac{-x}{3(x+3)}\cdot\frac{1}{x}$

$\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\sin x\right)$

$1\cdot 0 = \boxed{0}$

14) $\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\cot x}$

$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\frac{\cos x}{\sin x}}$

$\lim_{x\to\frac{\pi}{2}}\left(\cos x\cdot\frac{\sin x}{\cos x}\right) = \sin\frac{\pi}{2} = \boxed{1}$

15) $\lim_{x\to 0}\frac{(1-\cos x)^2}{x}$

$\lim_{x\to 0}\left(\frac{1-\cos x}{x}\right)(1-\cos x)$

$0\cdot(1-1) = \boxed{0}$

16) $\lim_{x\to\pi} x\sec x$

$= \pi\cdot\frac{1}{\cos\pi}$

$= \pi\cdot -1$ $\boxed{-\pi}$

10) $\lim_{\Delta x\to 0}\frac{2x+2\Delta x-2x}{\Delta x} = \lim_{\Delta x\to 0}\frac{2\Delta x}{\Delta x} = \lim_{\Delta x\to 0} 2 = \boxed{2}$