Logarithmic Differentiation

1. $y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x} \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \cdot \frac{1}{x} + \ln x \cdot \frac{-1}{x^2}$

$\frac{dy}{dx} = y\left(\frac{1}{x^2} - \frac{\ln x}{x^2}\right)$

$\frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$

2. $y = (\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 1$

$\frac{dy}{dx} = y\left(x \cot x + \ln(\sin x)\right)$

$\frac{dy}{dx} = (\sin x)^x \left(x \cot x + \ln(\sin x)\right)$

3. $y = x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x\left(\frac{1}{x}\right) + \ln x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = y\left(\frac{\ln x}{x} + \frac{\ln x}{x}\right)$

$\frac{dy}{dx} = x^{\ln x}\left(\frac{2\ln x}{x}\right)$

4. $y = (\ln x)^{\cos x}$

$\ln y = \cos x \ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) \cdot -\sin x$

$\frac{dy}{dx} = y\left(\frac{\cos x}{x \ln x} - \ln(\ln x) \cdot \sin x\right) = (\ln x)^{\cos x}\left(\frac{\cos x}{x \ln x} - \ln(\ln x) \cdot \sin x\right)$

5. $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$

$\frac{dy}{dx} = y\left(1 + \ln x\right) = x^x\left(1 + \ln x\right)$

6. $y = (\ln x)^x$

$\ln y = x \cdot \ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) \cdot 1$

$\frac{dy}{dx} = y\left(\frac{1}{\ln x} + \ln(\ln x)\right)$

$\frac{dy}{dx} = (\ln x)^x\left(\frac{1}{\ln x} + \ln(\ln x)\right)$

7. $y = (2x+1)^5 (x^4-3)^6$

$\ln y = 5\ln(2x+1) + 6\ln(x^4-3)$

$\frac{1}{y}\frac{dy}{dx} = 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4-3} \cdot 4x^3$

$\frac{dy}{dx} = y\left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}\right)$

$\frac{dy}{dx} = (2x+1)^5 (x^4-3)^6\left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}\right)$

8. $f(x) = \sqrt[4]{\frac{x^2+1}{x^2-1}}$

$\ln y = \frac{1}{4}\left[\ln(x^2+1) - \ln(x^2-1)\right]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{4}\left[\frac{1}{x^2+1} \cdot 2x - \frac{1}{x^2-1} \cdot 2x\right]$

$\frac{dy}{dx} = y \cdot \frac{1}{4}\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right] = \sqrt[4]{\frac{x^2+1}{x^2-1}}\left[\frac{x}{2(x^2+1)} - \frac{x}{2(x^2-1)}\right]$

$-2x$

$x^3 - x - x^3 - x$

$\frac{x(x^2-1) - x(x^2+1)}{2(x^2+1)(x^2-1)}$