

## More Practice with Laws of Logs

Expand each of the following as much as possible using laws of logarithms. When applicable, write "not possible".

1)  $\log\left(\frac{x^3 y^6}{\sqrt{z}}\right)$

2)  $\log\sqrt[4]{x^2 + y^2}$

3)  $\ln\left(\frac{x(x^2+1)}{\sqrt{x^2-1}}\right)$

$$3 \log x + 6 \log y - \frac{1}{2} \log z$$

$$\frac{1}{4} \log(x^2 + y^2)$$

$$\ln x + \ln(x^2+1) - \underbrace{\frac{1}{2} \ln(x^2-1)}_{\text{not possible}}$$

4)  $\log\left(\frac{x}{\sqrt[3]{1-x}}\right)$

5)  $\log^3\sqrt[3]{\frac{x+y}{x^6}}$

6)  $\ln(x-y)$

not possible

$$\log x - \frac{1}{3} \log(1-x)$$

$$\frac{1}{3} \log(x+y) - 2 \log x$$

Condense each of the following into a single logarithm using properties of logs:

7)  $6 \log x - 2 \log y + \frac{1}{3} \log t$

$$\log \frac{x^6 \sqrt[3]{t}}{y^2}$$

8)  $5 \log x - \frac{1}{4} \log(x^2+1) + 2 \log(x-1)$

$$\log \frac{x^5 (x-1)^2}{\sqrt[4]{x^2+1}}$$

9)  $\ln\left(\frac{a}{a^2-b^2}\right) + \ln\left(\frac{a-b}{a^2}\right)$

$$\ln\left(\frac{1}{a^2+ab}\right)$$

10)  $2(\log_5 x - 3 \log_5 z + 2 \log_5 y)$

$$\log_5 \frac{x^2 y^4}{z^6}$$

11)  $3 \ln x - \left(2 \ln y + \frac{1}{2} \ln z\right)$

$$\ln \frac{x^3}{y^2 \sqrt{z}}$$

12)  $4(2 \ln x - 3 \ln y) + 3(4 \ln y - \ln z)$

$$\ln(x^5)$$

13)  $\frac{1}{3} \log_4 x + 6 \log_4 y$

$$\log_4\left(\sqrt[3]{x} \cdot y^2\right)$$

14)  $\log(x+3) - (\log(x^2-9) - \log(x^3-27))$

$$\log(x^2+3x+9)$$

15)  $4 \ln x - \frac{1}{2}(6 \ln y - \frac{1}{4} \ln z)$

$$\ln \frac{x^4 \sqrt[8]{z}}{y^3}$$

Solve each of the following:

16)  $30 = 32(1 - 2^{-t})$

$$\frac{15}{16} = 1 - 2^{-t}$$

$$-\frac{1}{16} = -2^{-t}$$

$$\frac{1}{16} = 2^{-t}$$

$$2^{-4} = 2^{-t}$$

$$\boxed{t=4}$$

17)  $\log_3(x+2) = 4$

$$3^4 = x+2$$

$$81 = x+2$$

$$\boxed{x=79}$$

18)  $\frac{10}{1+e^{-x}} = 2$

$$10 = 2 + 2e^{-x}$$

$$8 = 2e^{-x}$$

$$4 = e^{-x}$$

$$\ln 4 = \ln e^{-x}$$

$$\ln 4 = -x$$

$$\begin{cases} x = -\ln 4 = \ln 4 \\ = \ln \frac{1}{4} \end{cases}$$

$$19) \log_5(x+1) - 2 = \log_5(x-1)$$

$$-2 = \log_5(x-1) - \log_5(x+1)$$

$$-2 = \log_5\left(\frac{x-1}{x+1}\right)$$

$$5^{-2} = \frac{x-1}{x+1}$$

$$\frac{1}{25} = \frac{x-1}{x+1}$$

$$x+1 = 25x - 25 \quad \boxed{x = \frac{13}{12}}$$

$$26 = 24x$$

$$22) \log_2(3x+2) = 3 + \log_2 x$$

$$\log_2(3x+2) - \log_2 x = 3$$

$$\log_2 \frac{3x+2}{x} = 3$$

$$2^3 = \frac{3x+2}{x}$$

$$8x = 3x+2$$

$$5x = 2 \quad \boxed{x = \frac{2}{5}}$$

$$25) \log_{\sqrt{216}} x = \frac{4}{3}$$

$$\sqrt{216}^{\frac{4}{3}} = x$$

$$\boxed{36 = x}$$

$$28) \left(\frac{1}{16}\right)^x = 64$$

$$(4^{-2})^x = 4^3$$

$$-2x = 3 \quad \boxed{x = -\frac{3}{2}}$$

$$20) 2 = \log_2(x^2 - x - 2)$$

$$2^2 = x^2 - x - 2$$

$$0 = x^2 - x - 6$$

$$0 = (x-3)(x+2)$$

$$\boxed{x = 3, x = -2}$$

$$21) \ln(x+4) = 3$$

$$e^3 = x+4$$

$$\boxed{x = e^3 - 4}$$

$$23) 2\log_5 x - \log_5 9 = 2$$

$$\log_5 x^2 - \log_5 9 = 2$$

$$\log_5 \frac{x^2}{9} = 2$$

$$5^2 = \frac{x^2}{9}$$

$$x^2 = 225$$

$$\boxed{x = 15} \rightarrow 15$$

$$24) \log(x^2) = \log 4 + \log 5$$

$$\log(x^2) = \log 100$$

$$x^2 = 20$$

$$x = \pm \sqrt{20}$$

$$26) \log_9 8 = \log_9 \frac{1}{2} + 2\log_9 x$$

$$\log_9 8 = \log_9 \frac{1}{2} + \log_9 x^2$$

$$\log_9 8 = \log_9 (\frac{1}{2}x^2)$$

$$8 = \frac{1}{2}x^2$$

$$16 = x^2$$

$$\boxed{x = 4} \rightarrow 4$$

$$27) 6e^{2x} + 45 = 3e^x$$

$$0 = 3e^{4x} - 6e^x + 45$$

$$3e^{4x}$$

$$0 = (3e^{2x} + 9)(e^{2x} - 5)$$

$$3e^{2x} + 9 = 0 \quad e^{2x} = 1$$

$$e^{2x} = -3 \quad 2x = \ln(-3) \quad \boxed{x = \frac{\ln(-3)}{2}}$$

$$29) 9^{2x} \cdot \left(\frac{1}{27}\right)^{x-1} = 81$$

$$30) \log_3(x-1) - \log_3(x+6) = \log_3(x-2) - \log_3(x+3)$$

$$(3^2)^{2x} \cdot (3^{-3})^{x-1} = 3^4$$

$$\log_3 \frac{x-1}{x+6} = \log_3 \frac{x-2}{x+3}$$

$$3^{4x+3x+3} = 3^4$$

$$\frac{x-1}{x+6} = \frac{x-2}{x+3}$$

$$x+3 = 4 \\ \boxed{x = 1}$$

$$\frac{(x+6)(x+3)(x-1)}{x+6} = \frac{(x-2)(x+6)(x+1)}{x+3}$$

$$x^2 + 2x - 3 = x^2 + 4x - 12$$

$$9 = 2x \\ \boxed{x = 9/2}$$

Evaluate:

31)  $\log_{25}\left(\frac{125}{\sqrt[3]{5}}\right)$

$$\frac{4}{3}$$

32)  $\log_{49}\left(\frac{1}{7}\right)$

$$-\frac{1}{2}$$

33)  $\log_8\left(\frac{2}{\sqrt[4]{4}}\right)$

$$\frac{1}{6}$$

34)  $\frac{\log_4 16}{\log_3\left(\frac{1}{27}\right)}$

$$-\frac{2}{3}$$

35)  $\log_3 2 \div \log_3 8$

$$\frac{\log_3 2}{\log_3 8} = \log_8 2$$

36)  $\log_2 3 - \log_2 12 = \log_2 \frac{1}{4}$

$$-2$$

37) Which of the following  $\frac{\log 27}{\log 3}$ ? (Circle all that apply)

$$= 3$$

a)  $\log 9$

b) 3

c)  $-\log 3^{-1}$

d)  $\log 24$

38) Which of the following are equivalent?

i.  $\frac{\log_6 216}{\log_6 36} = \frac{3}{2}$  ii.  $\log_6 \frac{216}{36} = 1$  iii.  $\log_6 216 - \log_6 36 = 1$

a) i & ii

b) ii & iii

c) iii

d) none of these

e) all of these

39) Which of the following are equivalent?

i.  $\frac{1}{3} \log 270$  ii.  $\log 90 = \log 9 + \log 10$  iii.  $\frac{1}{3} + \log 3$   
 $= \frac{1}{3} (\log(27 \cdot 10))$   $= (\log 9) + 1$

a) i & ii

b) i & iii

c) ii & iii

d) none of these

e) all of these

40) Given  $\log 7 = x$ ,  $\log 5 = y$ ,  $\log 3 = z$  determine each of the following:

a)  $\log 9$

$$\log 3^2 = 2 \log 3 = \boxed{2z}$$

b)  $\log 150$

$$\begin{aligned} &\log(5 \cdot 3 \cdot 2) \\ &\log 5 + \log 3 + \log 2 \\ &\boxed{2y + z + \log 2} \end{aligned}$$

c)  $\log 7$

$$\frac{\log 7}{\log 5} = \boxed{x}$$

d)  $\log_7 15$

$$\begin{aligned} \frac{\log 15}{\log 7} &= \log 5 + \log 3 \\ &= \frac{\log 7}{x} \\ &= \boxed{y+z} \end{aligned}$$

e)  $\log(3/5)$

$$\begin{aligned} &\log 3 - \log 5 \\ &\boxed{z-y} \end{aligned}$$

f)  $\log 30$

$$\begin{aligned} &\log(2 \cdot 3 \cdot 5) \\ &= \log 2 + \log 3 + \log 5 \\ &= \boxed{(\log 2) + z + y} \end{aligned}$$

41) State the domain of each of the following:

a)  $f(x) = \ln(9 - x)$

$$9 - x > 0$$

$$-x > -9$$

$$x < 9 \quad (-\infty, 9)$$

b)  $f(x) = \ln(3x + 2)$

$$3x + 2 > 0$$

$$3x > -2$$

$$x > -\frac{2}{3} \quad \left(-\frac{2}{3}, \infty\right)$$

42) State the transformations applied to the graph of  $f(x)$  which result in the graph of  $g(x)$ .

a)  $f(x) = \left(\frac{4}{5}\right)^x$

$$g(x) = 3\left(\frac{5}{4}\right)^{x+3} = 3\left(\frac{4}{5}\right)^{-1(x+3)}$$

left 3 units

reflect over y-axis

vertical stretch \*3

b)  $f(x) = 3^x$

$$g(x) = \frac{1}{5} \cdot 9^{x-1} + 4 = \frac{1}{5}(3)^{2(x-1)} + 4$$

right 1 unit

horiz. shrink \* $\frac{1}{2}$

vert. shrink \* $\frac{1}{5}$

up 4 units

c)  $f(x) = 2^x$

$$g(x) = -3\left(\frac{1}{4}\right)^{2-x} - 1 = -3(2)^{2(2-x)} - 1 = -3(2)^{2(x-2)} - 1$$

right 2 units

horiz. shrink \* $\frac{1}{2}$

vert. stretch \*3

reflect over x-axis  
down 1

d)  $f(x) = e^x$

$$g(x) = -e^{6-3x} = -e^{-3(x-2)}$$

right 2 units

horiz. shrink \* $\frac{1}{3}$

refl. over x-axis

refl. over y-axis

43) Given that  $f(x) = \ln(x+3)$  determine each of the following:

a)  $f(-3) = \underline{\text{not possible}}$

b)  $f(-2) = \underline{\ln 1} = \underline{0}$

c)  $f(2) = \underline{\ln 5}$

d)  $f(2x) = \underline{\ln(2x+3)}$

d)  $f(x^2 - 3) = \underline{\ln(x^2)} \\ = 2 \ln x$

e)  $f(x^2 - 12) = \underline{\ln(x^2-9)} \\ = \ln(x+3) + \ln(x-3)$

f)  $f(x^2 + 6x + 6) = \underline{\ln(x^2+6x+9)} \\ = \ln[(x+3)(x+3)] \\ = \ln(x+3)^2 \\ = 2 \ln(x+3)$

44) Now...using your answers from #43 determine whether each of the following is TRUE or FALSE:

a)  $f(-3) = 0$

False

b)  $f(-2) = 0$

True

c)  $f(2x) = \underline{\ln 5} + \underline{\ln(x+3)}$

False

d)  $f(x^2 - 3)$  has a domain  
of all real numbers False

$$\ln(x^2) = \underline{2 \ln(x)} \\ D: (0, \infty)$$

e)  $f(x^2 - 12) = \underline{\ln(x+3)} + \underline{\ln(x-3)}$

True

f)  $\frac{1}{2}f(x^2 + 6x + 6) = f(x)$

$$\frac{1}{2}(2 \cdot \ln(x+3)) = \ln(x+3) \\ \text{True}$$