

ICM Logarithmic Functions Worksheet #2

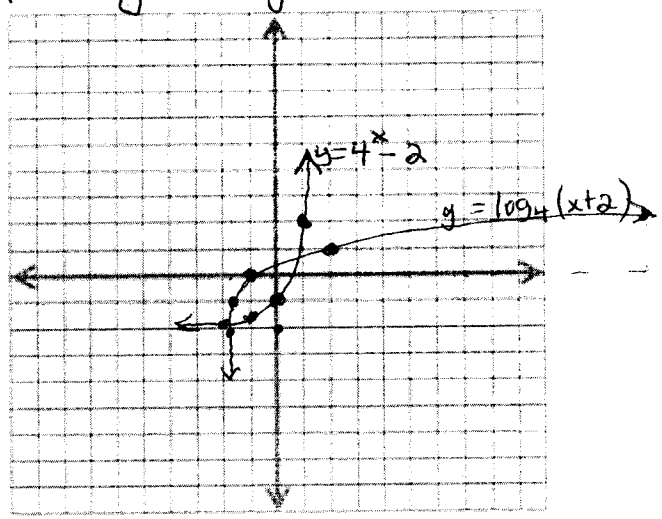
1. Sketch the graph of each function in the same coordinate plane. Then give the requested information for each graph.

a.  $f(x) = 4^x - 2$

x	f(x)
-2	-1.9375
-1	-1.75
0	-1
1	2
2	14

b.  $g(x) = \log_4(x+2) \rightarrow 4^x = y+2 \quad y = 4^x - 2$   
 $x = \log_4(y+2)$

x	g(x)
Switch	
x & y	
Coordinates	



f(x): domain  $(-\infty, \infty)$  range  $(-2, \infty)$  x-intercept  $(\frac{1}{2}, 0)$   
 y-intercept  $(0, -1)$  horizontal asymptote  $y = -2$   
 g(x): domain  $(-2, \infty)$  range  $(-\infty, \infty)$  x-intercept  $(-1, 0)$   
 y-intercept  $(0, \frac{1}{2})$  horizontal asymptote none

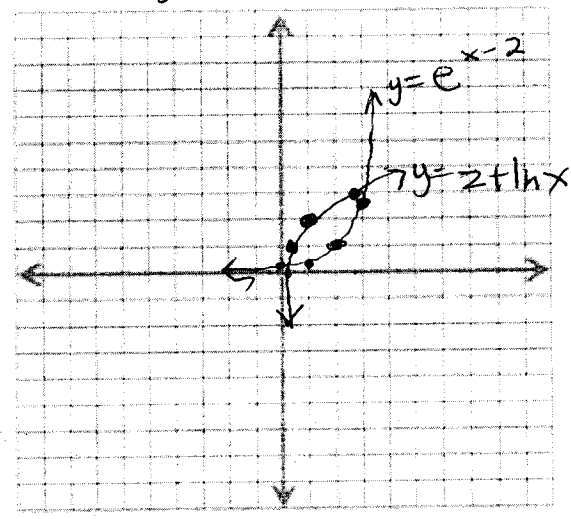
c.  $f(x) = e^{x-2}$

x	f(x)
-2	.0183
-1	.0498
0	.135
1	.368
2	1
3	2.718

d.  $g(x) = 2 + \ln x$   
 $x = 2 + \ln y$

x	g(x)
Switch	
x & y	
Coordinates	

$x-2 = \ln y \quad y = e^{x-2}$



f(x): domain  $(-\infty, \infty)$  range  $(0, \infty)$  x-intercept none  
 y-intercept  $(0, .135)$  horizontal asymptote  $y = 0$   
 g(x): domain  $(0, \infty)$  range  $(-\infty, \infty)$  x-intercept  $(.135, 0)$   
 y-intercept none horizontal asymptote none

2. Find the exact value of each logarithm without using a calculator.

1.  $\log_2 1 = 0$

2.  $\log_8 8 = 1$

3.  $\log_5 25 = 2$

4.  $\log_3(\frac{1}{9}) = -2$

5.  $\log_{\frac{1}{2}} 16 = -4$

6.  $\log_{\frac{1}{3}} 9 = -2$

7.  $\log_{10} \sqrt{10} = \frac{1}{2}$

8.  $\log_5 \sqrt[3]{25} = \frac{2}{3}$

9.  $\log_{\sqrt{2}} 4 = 4$

10.  $\log_{\sqrt{3}} 9 = 4$

11.  $\ln \sqrt{e} = \frac{1}{2}$

12.  $\ln e^3 = 3$

5.  $(2^{-1})^y = 16$   
 $2^{-y} = 2^4$

6.  $(3^{-1})^y = 3^2$   
 $-y = 2$

9.  $(2^{1/2})^y = 2^2$   
 $\frac{1}{2}y = 2$

10.  $(3^{1/2})^y = 3^2$   
 $\frac{1}{2}y = 2$   
 $y = 4$

3. Using  $y = \ln x$  as the parent function, describe the transformations to obtain each graph.

a.  $y = 8 - \ln x$  reflect over x-axis  
 shift up 8

b.  $y = \ln(-x) - 4$  reflect over y-axis  
 shift down 4

c.  $y = 3 + 2 \ln x$  vertical stretch by a factor of 2  
 shift up 3

d.  $y = 6 - \ln(x - 9)$  horiz. shift rt. 9  
 refl over x-axis

e.  $y = -1 - \frac{1}{5} \ln(x + 2)$  shift up 6  
 horiz. shift left 2  
 vert. shrink by a factor of  $\frac{1}{5}$   
 refl over x-axis, shift down 1

4. The pH of a chemical solution is given by the formula  $\text{pH} = -\log_{10} [H^+]$  where  $[H^+]$  is the concentration of hydrogen ions in moles per liter. Values of pH range from 0 (acidic) to 14 (alkaline).

- a. Find the pH of a 1 liter container of water with 0.0000001 mole of hydrogen ion.
- b. Find the hydrogen ion concentration of a mildly acidic solution with a pH of 4.2.

a.  $-\log_{10} (0.0000001) = 7$

b.  $4.2 = -\log_{10} [H^+]$   $H^+ = 6.3096 \times 10^{-5}$   
 $H^+ = 0.000063096$

5. The number of watts  $w$  provided by a space satellite's power supply after  $d$  days is given by the formula  $w = 50e^{-0.004d}$ .

- a. How long will it take for the available power to drop to 30 watts?  $30 = 50e^{-0.004d}$
- b. How long will it take for the available power to drop to only 5 watts?  $5 = 50e^{-0.004d}$

a. 127.706 days  
 b. 575.646 days

6. If a single pane of glass obliterates 10% of the light passing through it, then the percent  $P$  of the light that passes  $n$  successive panes is given approximately by the equation  $P = 100e^{-0.1n}$ .

- a. How many panes are necessary to block at least 50% of the light?  $50 = 100e^{-0.1n}$
- b. How many panes are necessary to block at least 75% of the light?  $75 = 100e^{-0.1n}$

a. 6.93  $\Rightarrow$  7 panes  
 b. 2.87  $\Rightarrow$  3 panes