ICM Properties of Logarithms Worksheet #3

KEY

1. Write each expression as a sum and/or difference of logarithms. Express powers as factors.
   a. $\ln(x^3\sqrt{1-x})$ $3\ln x + \frac{1}{2}\ln(1-x)$
   b. $\log_4\left(\frac{x^2}{x-3}\right)$ $2\log_4 x - \log_4(x-3)$

2. Write each expression as a single log.
   a. $3\log_5 u + 4\log_5 v$ $\log_5(u^3v^4)$
   b. $\log_{\frac{1}{2}}\sqrt{x} - \log_{\frac{1}{2}}x^3$ $\log_{\frac{1}{2}}\left(\frac{\sqrt{x}}{x^3}\right) = \log_{\frac{1}{2}}(x^{-5/2}) = -\frac{5}{2}\log_{\frac{1}{2}}x$

3. Use the change-of-base formula to evaluate each logarithm. Round to 3 decimal places.
   a. $\log_3 21$ 2.771
   b. $\log_{\frac{1}{3}}15$ −2.465

4. Solve each equation. Show all algebraic work on another sheet of paper.
   a. $\log_2(2x+1) = 3$
   b. $\log_3(x^2+1) = 2$
   c. $2\log_5 x = 3\log_5 4$
   d. $\log x + \log(x+15) = 2$
   e. $2^{2x+1} = 4$
   f. $3^{x^3} = 9^x$
   g. $4^x = 8$
   h. $2^x = 10$
   i. $2^{x+1} = 5^{1-2x}$
   j. $400e^{0.2x} = 600$
   k. $\log_2(x-1) - \log_2(x+6) = \log_2(x-2) - \log_2(x+3)$

5. Use a calculator to solve each equation.
   a. $\log_5 x + \log_3 x = 1$ 1.921
   b. $e^{2x} = x + 2$ −1.980 and .448
   c. $\ln x = x^3 - 1$ .390 and 1

a. $\log_2(2x+1) = 3$

$2x+1 = 2^3$

$2x+1 = 8$

$2x = 7$

$\boxed{x = \frac{7}{2}}$

b. $\log_3(x^2+1) = 2$

$x^2+1 = 3^2$

$x^2 = 8$

$x = \pm\sqrt{8} = \boxed{\pm 2\sqrt{2}}$

c. $2\log_5 x = 3\log_5 4$

$\log_5 x^2 = \log_5 4^3$

$x^2 = 4^3$

$x = \pm\sqrt{64}$

$\boxed{x = 8}$

d. $\log(x(x+15)) = 2$

$x(x+15) = 10^2$

$x^2+15x-100 = 0$

$(x+20)(x-5) = 0$

$x = -20$ $\boxed{x = 5}$

e. $2^{2x+1} = 4$

$2^{2x+1} = 2^2$

$2x+1 = 2$

$\boxed{x = 1/2}$

f. $3^{x^3} = 9^x$

$3^{x^3} = (3^2)^x \rightarrow 3^{2x}$

$x^3 = 2x$

$x^3 - 2x = 0$

$x(x^2-2) = 0$

$\boxed{x = 0,\ x = \pm\sqrt{2}}$

g. $4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

h. $2^x = 10$

$\log 2^x = \log 10$

$x \cdot \log 2 = \log 10$

$\boxed{x = \frac{\log 10}{\log 2} \approx 3.322}$

i. $2^{x+1} = 5^{1-2x}$

$(x+1)\ln 2 = (1-2x)\ln 5$

$x\cdot\ln 2 + \ln 2 = \ln 5 - 2x\ln 5$

$x\ln 2 + 2x\ln 5 = \ln 5 - \ln 2$

$x(\ln 2 + 2\ln 5) = \ln 5 - \ln 2$

$\boxed{x = \frac{\ln 5 - \ln 2}{\ln 2 + 2\ln 5} \approx .234}$

j. $400e^{0.2x} = 600$

$e^{0.2x} = 1.5$

$0.2x \cdot \ln e = \ln 1.5$

$0.2x = \ln 1.5$

$$\boxed{x = \frac{\ln 1.5}{0.2} \approx 2.027}$$

k. $\log_2\left(\frac{x-1}{x+6}\right) = \log_2\left(\frac{x-2}{x+3}\right)$

$$\frac{x-1}{x+6} = \frac{x-2}{x+3}$$

$$\frac{x-1}{x+6} - \frac{x-2}{x+3} = 0$$

$$\frac{(x-1)(x+3)-(x-2)(x+6)}{(x+6)(x+3)} = 0$$

$$\frac{x^2+2x-3-x^2-4x+12}{(x+6)(x+3)} = 0$$

$$\frac{-2x+9}{(x+6)(x+3)} = 0$$

$-2x+9=0$

$$\boxed{x = \frac{9}{2}}$$