# Logistic Growth

1. The population P(t) of a species satisfies the logistic differential equation, $\frac{dP}{dt} = P(2 - \frac{P}{5000})$, where the initial population is P(0) = 3000 and t is the time in years. What is the $\lim_{t \to \infty} P(t)$?

   a. 2500  b. 3000  c. 4200  d. 5000  e. 10,000

   $\frac{dp}{dt} = 2P\left(1 - \frac{P}{10000}\right)$

2. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9 AM.

   $\frac{dp}{dt} = 6P\left(1 - \frac{P}{2000}\right)$  $k = 6$  $L = 2000$

   a. How many students have heard the rumor when it is spreading the fastest? $\boxed{1000}$
   b. If P(0) = 5, solve for P as a function of t.
   c. Use you answer in part b to determine how many hours have passed when half the student body has heard the rumor.

   b) $b = \frac{2000}{5} - 1 = 399$  $\boxed{p(t) = \frac{2000}{1 + 399e^{-6t}}}$

   c) $1000 = \frac{2000}{1 + 399e^{-6t}}$  $\boxed{t = .998 \text{ hr}}$

3. Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks. What is the carrying capacity?

   $\frac{dp}{dt} = .05P\left(1 - \frac{P}{100}\right)$

   $\boxed{100}$

4. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt} = P\left(3 - \frac{P}{2000}\right)$. What is $\lim_{t \to \infty} P(t)$? What does the limit represent in the context of this problem?

   $\frac{dp}{dt} = 3P\left(1 - \frac{P}{6000}\right)$  $\boxed{\lim_{t \to \infty} P(t) = 6000 \text{ ; \# of people at a dance}}$

5. Suppose you are in charge of stocking a fish pond with fish for which the rate o population growth is modeled by the differential equation $\frac{dP}{dt} = 0.08P - 0.0002P^2$

   $\frac{dp}{dt} = 0.08P\left(1 - \frac{P}{400}\right)$  $k = .08$  $L = 400$

   a. If P(0) = 50, then $\lim_{t \to \infty} P(t) =$ 400.
   b. If P(0) = 300, then $\lim_{t \to \infty} P(t) =$ 400.
   c. If P(0) = 500, then $\lim_{t \to \infty} P(t) =$ 400.

6. A certain national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park presently. The population growth of bears can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.1P - 0.001P^2$, where t is measured in years.

   $\frac{dp}{dt} = 0.1P\left(1 - \frac{P}{100}\right)$  $k = 0.1$  $L = 100$

   a. Solve for P as a function of t
   b. Use the solution found in part a to find the number of bears in the park when t = 3 years. $\boxed{13 \text{ bears}}$
   c. Use the solution found in part a to find how many years it will take for the bear population to reach 50 bears.

   a) $b = \frac{100}{10} - 1 = 9$

   $\boxed{p(t) = \frac{100}{1 + 9e^{-0.1t}}}$

   c) $50 = \frac{100}{1 + 9e^{-\cancel{.1t}}}$

   $\boxed{t = 21.972 \text{ yrs}}$

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