

Show your work for each problem.

1. Using 20th century US Census data, the population of New York state can be modeled by $P(t) = \frac{19.875}{1 + 57.993e^{-0.035005t}}$ where P is the population in millions and t is the number of years since 1800. Based on this model,



- a) what was the population of NY in 1850? 1.795 million
- b) what will NY's population be in 2010? 19.162 million
- c) what is NY's maximum sustainable population (limit to growth)? 19.875 million

2. The logistic growth model $P(t) = \frac{0.9}{1 + 3.5e^{-0.339t}}$ relates the proportion of new personal computers sold at Best Buy that have Intel's latest coprocessor t months after it has been introduced.

- a) What proportion of new personal computers sold at Best Buy will have Intel's latest coprocessor when it is first introduced (when t = 0)? 20%
- b) Determine the maximum proportion of new personal computers sold at Best Buy that will have Intel's latest coprocessor. 90%
- c) When will 75% of new personal computers sold at Best Buy have Intel's latest coprocessor? 8.443 months

3. The logistic model $P(t) = \frac{1000}{1 + 32.33e^{-0.439t}}$ relates to the population of a bacteria after t hours.

- a) What is the carrying capacity of the environment? 1000 bacteria
- b) What was the initial amount of bacteria in the population? 30 bacteria
- c) When will the amount of bacteria be 800? 11.076 hrs

4. Write a logistic growth model of the form $f(x) = \frac{c}{1 + ae^{-bx}}$ with the following characteristics:

$f(0) = 20$, $f(3) = 120$, and $\lim_{x \rightarrow \infty} f(x) = 500$ (circled and labeled "carrying cap.")

$$f(x) = \frac{500}{1 + 24e^{-0.675x}}$$

5. Write a function of time, h(t), where the initial height = 15 cm, the limit to growth = 120 cm, and at t = 2 years, its height = 18.5 cm

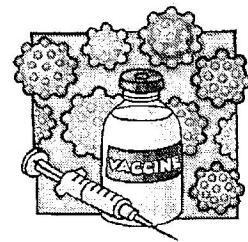
$$h(t) = \frac{120}{1 + 7e^{-0.122t}}$$

6. The function $f(t) = \frac{30000}{1+20e^{-1.5t}}$ describes the number of people, $f(t)$, who have become ill with the flu t weeks after its initial outbreak in a town with 30,000 inhabitants.

a) How many people are ill with the flu when the outbreak began? **1428**

b) How many people were sick by the end of the third week? **24546**

c) What is the limiting size of $f(t)$, the population that becomes ill? **30000**



7. The logistic growth function $f(t) = \frac{500}{1+83.3e^{-0.162t}}$ describes the population of endangered species of birds t years after they are introduced to a non-threatening habitat.

a) How many birds were initially introduced to the habitat? **5.931**

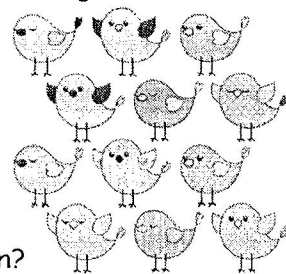
⇒ 5 birds

b) How many birds are expected in the habitat after 10 years?

$f(10) = 28.596$ ⇒ 28 birds

c) What is the limiting size of the bird population that the habitat will sustain?

500 birds



8. Europe's Great Plague of 1666 devastated Eyam, England. There were 261 people in the village; only 83 survived. The logistic growth function $f(t) = \frac{171}{1+18.6e^{-0.0747t}}$ models the number of people in Eyam who were infected t days after the outbreak.

a) How many people were infected when the outbreak began? **8**

b) How many people were infected after 45 days? **103**

c) According to the model, what is the limiting size of Eyam's population that can become infected? **171**

9. A company introduces a new software product on a trial run in a city. They advertise the product on television and found the following data relating the percent P of people who bought the product after x ads were run.

a) Calculate the logistic model.

$y = \frac{82.294}{1+102.499e^{-0.038x}}$

b) What was the percent of people who bought the product when it was just introduced (that is, when $x = 0$)?

• 79.5 using the model • 2 using the chart

c) Use the model to determine the limiting percent of people who would buy the software (carrying capacity).

82.294%

d) How many ads had been run before 75% of people bought the product? **79.535 ⇒ 79 ads**

Number of Ads, x	People Who Bought, P (in %)
0	0.2
10	0.7
20	2.7
30	9.2
40	20.5
50	37.6
60	53.3
70	64.8
80	76.5
90	79.6