Calculus AB

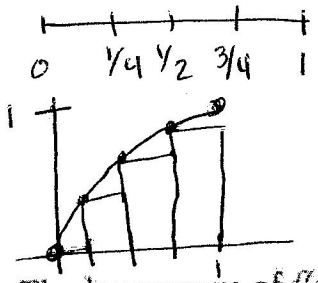
Homework – Riemann Sums

Multiple-Choice Questions: A graphing calculator is required for some questions.

1. The lower sum of $f(x) = \sqrt{x}$ on the interval [0, 1] with four equal subintervals is
   (A) 0.25
   (B) 0.518
   (C) 0.667
   (D) 0.768
   (E) 3.073

$\frac{1}{4}\left(0 + \frac{1}{2} + \sqrt{\frac{1}{2}} + \sqrt{\frac{3}{4}}\right)$

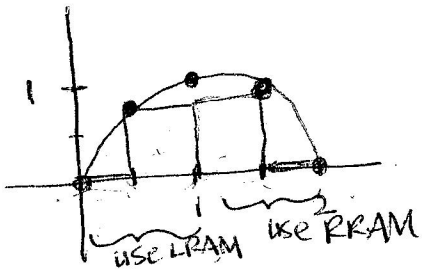
x	f(x)
0	0
1/4	1/2
1/2	$\sqrt{1/2}$
3/4	$\sqrt{3/4}$
1	1



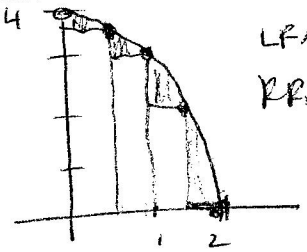
2. The lower sum of $f(x) = -(x-1)^2 + 1$ on the interval [0, 2] with four equal subintervals is
   (A) $\frac{1}{2}$
   (B) $\frac{3}{4}$
   (C) $\frac{4}{3}$
   (D) $\frac{3}{2}$
   (E) 2

LRAM: $\frac{1}{2}(0 + .75) = .375$

RRAM: $\frac{1}{2}(.75 + 0) = .375$

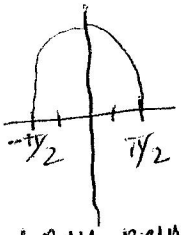


3. Which of the following is true for $f(x) = 4 - x^2$ on the interval [0, 2] with n equal subintervals?
   (A) the left-hand sum = the right-hand sum
   (B) the left-hand sum > the right-hand sum
   (C) the left-hand sum < the right-hand sum
   (D) the area under the curve > the left-hand sum
   (E) the area under the curve < the right-hand sum



LRAM – circumscribed
RRAM – inscribed

4. Which of the following is true for $f(x) = \cos x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ using four equal subintervals?
   (A) left-hand sum < right-hand sum
   (B) right-hand sum < left-hand sum
   (C) midpoint sum < left-hand sum
   (D) midpoint sum = left-hand sum
   (E) left-hand sum = right-hand sum

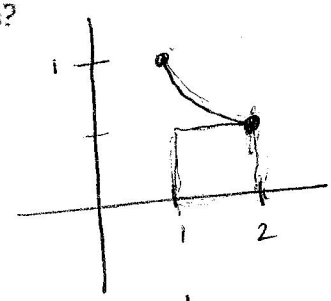


LRAM=RRAM

5. Which value of n gives the largest upper sum for $f(x) = 2x^2$ on the interval [−1, 1]?
   (A) $n = 4$
   (B) $n = 10$
   (C) $n = 20$
   (D) $n = 30$
   (E) $n = 50$

biggest approximation
⇒ use fewer rect

6. Which of the following is true for the lower sum of $f(x) = \frac{1}{x}$ on the interval [1, 2] for any number of subintervals?
   (A) lower sum > ln 2
   (B) lower sum ≥ $\frac{1}{2}$
   (C) lower sum = ln 2
   (D) lower sum = $\frac{1}{2}$
   (E) lower sum < 0.5



RRAM = lower sum
lowest lower sum
uses 1 rect
width = 1
ht = 1/2
1 · 1/2 = 1/2

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Free-Response Questions: A graphing calculator is required for some questions.

1. During a recent snowfall, several students monitored the accumulation of snow on the flat roof of their school. The table records the data they collected for the 12-hour period of the snowfall.

Number of Hours	Rate of Snowfall (in./hour)
0	0
2	1.5
3	2.1
4.5	2.4
6.5	2.8
8	2.2
10.5	1.8
12	1.6

(a) Use a right-hand sum to approximate the total depth of snow in the 12-hour period.

(b) Using the right-hand sum approximation, estimate the average rate of snowfall in the 12-hour period.

a) $2(1.5) + 1(2.1) + 1.5(2.4) + 2(2.8) + 1.5(2.2) + 2.5(1.8) + 1.5(1.6)$

$= \boxed{24.5 \text{ inches}}$

b) $\dfrac{\Delta \text{ amount}}{\Delta \text{ time}} = \dfrac{24.5}{12} = \dfrac{49}{24} \approx \boxed{2.042 \text{ in/hr}}$

2. A car slows down as it approaches a red light at an intersection. When the light turns green, the velocity of the car increases as shown in the table.

Time t (sec)	Velocity v (ft/sec)
0	8
2	14
4	22
6	30
8	40
10	45

(a) Find the average rate of change of the velocity v on the interval [0, 10].

(b) Approximate the distance traveled in the first ten seconds using five equal subintervals.

(c) Approximate the acceleration of the car at $t = 6$.

$(0, 8)\ (10, 45)$

a) $\dfrac{\Delta \text{ vel}}{\Delta \text{ time}} = \dfrac{45-8}{10-0} = 3.7$

$\boxed{3.7 \text{ ft/sec}^2}$

b) LRAM: $2(8+14+22+30+40)$

$= \boxed{228 \text{ ft.}}$

RRAM: $2(14+22+30+40+45)$

$= \boxed{302 \text{ ft.}}$

c) $(4, 22)(8, 40)$

$\dfrac{40-22}{8-4} = \dfrac{18}{4} = \boxed{4.5 \text{ ft/sec}^2}$

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