ICM Midterm Review Fall 2017

Election Theory

1. If 1543 people voted in the most recent local elections, how many votes are needed for a majority? 772

2. Determine the winner using the following methods:
   A. plurality B
   B. Borda C
   C. run-off A
   D. sequential run-off B
   E. approval (assume the top 2 preferences are "approved" of by each voter) C

	15	6	10	12	11
1st	A	B	B	C	D
2nd	B	D	C	A	C
3rd	C	A	D	D	A
4th	D	C	A	B	B

3. Use Condorcet's method to determine a winner:

   B

	5	7	4	6
1st	A	B	C	B
2nd	B	C	A	C
3rd	C	A	B	A

4. For the voting situation [8: 5, 4, 3, 1], find:
   A. the coalitions & their weights (see work)
   B. the winning coalitions AB, AC, ABC, ABD, ACD, BCD, ABCD
   C. Banzhaf Power Index for each voter A = 5/12 B = 3/12 C = 3/12 D = 1/12
   D. Shapley-Shubik Power Distribution for each voter A = 41.7% B = 25% C = 25% D = 8.3%

Apportionment

5. Suppose that HSHS will elect 20 members for the student government. Apportion the seats using the Hamilton and the Jefferson Methods.

Class	Size	Hamil	Jeff.
Seniors	400	4	4
Juniors	425	5	5
Sophomores	475	5	5
Freshmen	525	6	6

6. Three waitresses are to divide $75 in tips (all ones) based on the number of diners they served. Use Webster's Method to apportion the tips.

waitress	diners	Webster
Marcia	8	15
Jan	21	39
Cindy	11	21

Fair Division

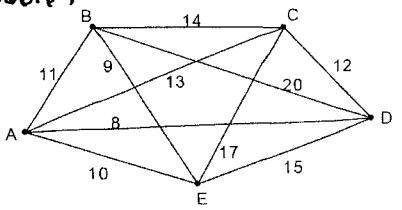
7. Use the method of sealed bids to divide up the estate:

Item	Andrew's bids	Billy Joe's bids	Carmelo's bids
cabin	$ 175,000	$ 180,000	$ 160,000
boat	$ 15,000	$ 18,500	$ 16,000
automobile	$ 20,000	$ 17,500	$ 16,000

Andrew: auto & $54166.67 Billy Joe: boat, cabin & pays $122,333.33
Carmelo: $68166.67

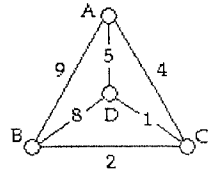
Graph Theory

8. Use the Nearest Neighbor algorithm to find a circuit starting at A. List the circuit and its weight.



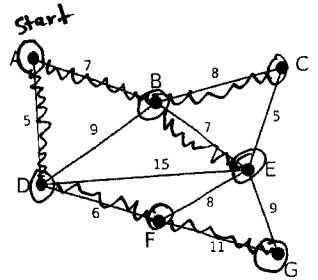
ADCBEA = 53

9. Apply the Brute Force algorithm to find the circuit of least distance starting at A. List the circuit and its weight.



ABCDA 17
ADCBA

10. Apply the Shortest Path algorithm starting at vertex A. What is the shortest path from A to G? What is the length of that path?



ADFG = 22

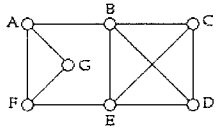
11. A tropical fish hobbyist had six different types of fish: Alphas, Betas, Certas, Deltas, Epsilons, and Fetas, which shall henceforth be designated by A, B, C, D, E, and F, respectively. Because of predator-prey relationships, water conditions, and size, some fish cannot be kept in the same tank. The following table shows which fish cannot be together:

Type	A	B	C	D	E	F
Cannot be with	B, C	A, C, E	A, B, D, E	C, F	B, C, F	D, E

What is the smallest number of tanks needed to keep all the fish? 3

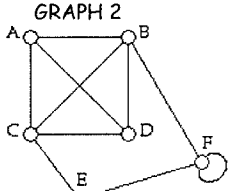
12.

GRAPH 1



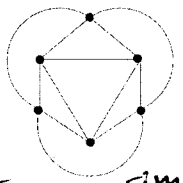
Simple

GRAPH 2



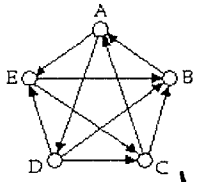
pseudograph

GRAPH 3



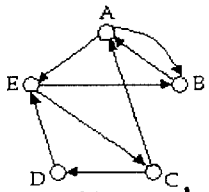
simple

GRAPH 4



directed

GRAPH 5



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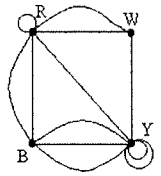
GRAPH 6



pseudograph

A. Classify each type of graph for #1-6.

B. Specify which of the graphs in #1-3 has an Euler circuit, an Euler path, or neither? #1 #3 #2

C. Does graph #1 have a Hamilton circuit, a Hamilton path, or neither?

D. For graph #6, find the degree of each vertex. B=5 R=7 W=3 Y=9

E. How many edges does graph #6 have? 12

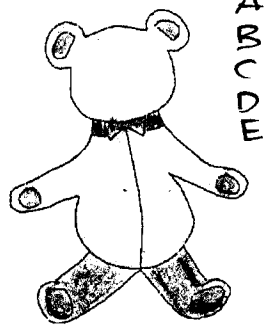
F. For graph #4, find the in-degree and out-degree of each vertex.

vertex	in	out
A	2	2
B	3	1
C	2	2
D	1	3
E	2	2

G. For graph #5, give an adjacency matrix.

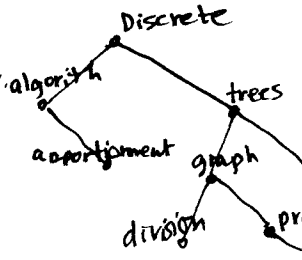
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

13. Color and find the chromatic number of the map to the right:



4

14. Construct a binary search tree in alphabetical order for following the words: discrete, algorithm, trees, apportionment, vote, graph, probability, statistics, division



15. For the tree, find the:

A. root F

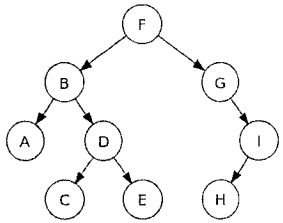
B. leaves A, C, E, H

C. children of B A, D

D. parent of G F

E. ancestors of E D, B, F

F. descendants of B C, D, E, A



Probability

16. 10 people want to be on the dance committee. How many 3 person committees are possible? 120

17. For a short trip you have packed 2 pairs of pants, 5 shirts and 3 sweaters in coordinating colors. How many outfits can you create if an outfit consists of pants, shirt and sweater? 30

18. 40 people apply for 5 sales positions at the mall.

A. If the 5 positions are identical, in how many ways can the vacancies be filled? 658,008

B. If the 5 positions are all different, in how many ways can the vacancies be filled? 78,960,960

19. In a certain state, license plates consist of 3 letters followed by 3 digits (0 to 9) followed by a letter. How many license plates can be formed if numbers and letters may not repeat? 258,336,000

20. One card is chosen at random from a standard deck of 52 playing cards. Find each of these probabilities:

A. p(Jack) $\frac{1}{13}$

B. p(black card) $\frac{1}{2}$

C. p(ace of spades) $\frac{1}{52}$

D. p(8,9 or 10) $\frac{3}{13}$

E. p(face or queen) $\frac{3}{13}$

F. p(red and 2) $\frac{1}{26}$

21. A fair die is rolled 200 times.

A. About how many even numbers would you expect? 100

B. About how many fours would you expect? $33\frac{1}{3}$

22. Grace rolled a die 1500 times. The results are shown in the chart below.
   A. What is the experimental probability of rolling a "3"? $\frac{1}{5}$
   B. What is the theoretical probability of rolling a "3"? $\frac{1}{6}$
   C. Using the experimental results, what is the odds in favor of rolling a "3"? 300:1200 = 1:4
   D. Using the experimental results, what is the odds against rolling a "3"? 1200:300 = 4:1

# on die	1	2	3	4	5	6
Times rolled	230	245	300	280	215	230

23. Use the information contained in the chart from a survey of students to answer these questions.

	male	female
likes apples	25	75
doesn't like apples	50	150

   A. What is the probability that a person who doesn't like apples is female? $\frac{3}{4}$
   B. What is the probability that a female doesn't like apples? $\frac{2}{3}$
   C. What is the probability that a student likes apples? $\frac{1}{3}$

24. A student is applying to Harvard and Dartmouth. He estimates that he has a probability of 0.5 of being accepted at Dartmouth and 0.3 of being accepted at Harvard. He further estimates the probability that he will be accepted by both is 0.2. What is the probability that he is accepted by Dartmouth given that he has been accepted by Harvard? $\frac{2}{3}$

25. The Mu Alpha Theta Club at HSHS is sponsoring a booth at the school carnival. A player selects a colored duck from a pond in order to win a prize. The pond contains the following number and color of ducks: 20 white, 15 gray, 6 red, 3 green, 3 blue, and 1 yellow. You are blindfolded.
   A. You select one duck. What is the probability that the duck will be gray or yellow? $\frac{1}{3}$
   B. You select two ducks in succession without replacement. What is the probability you select a green duck and then a white duck? $\frac{5}{188}$
   C. You select two ducks in succession without replacement. What is the probability you select a 2 red ducks? $\frac{5}{376}$
   D. You select one duck, show it to the game attendant, put it back, and select another duck. What is the probability you select a green duck and then a white duck? $\frac{5}{192}$

26. A manufacturing company has 2200 employees. The employees attend seminars to obtain additional training. Suppose that 625 employees will attend the January seminar, 850 will attend the February seminar, and 500 will attend both seminars. What is the probability that an employee selected at random will be attending the January or February seminar? $\frac{39}{88}$

27. I deal three cards in succession from a shuffled deck. What's the probability that the first card is the queen of spades, the second is the six of diamonds, and the third is a queen? $\frac{1}{44200}$

28. Suppose thirteen percent of the population is left-handed. If we select 5 people at random, find the probability that:
   A. there are exactly 3 lefties in the group. .017
   B. there are at least 2 lefties in the group. .129
   C. there are no more than 2 lefties in the group. .982
   D. there are less than 3 lefties in the group .982
   E. How many lefties would you expect to be in a group of 150 people? 19.5

29. A basketball player makes 80% of her shots. Find the probability in tonight's game, she
   A. makes her first basket on her 4th attempt .0064
   B. misses for the first time on her 5th shot attempt .08192
   C. makes her first shot on one of her first 3 attempts .992
   D. What is the expected number of shots before she misses? 5

<u>Statistics</u>

30. We can assume that physical coordination is normally distributed (i.e., few people are clumsy and few are professional dancers/athletes). The National standard level of physical coordination (PC) has a mean of 50 with a standard deviation of 12. Answer the following regarding 175 Americans.
   A. How many would you expect to have PCs of 70 or higher? 8.363
   B. How many would you expect to have PCs lower than 40? 35.407
   C. How many would have PCs ranging from 25 - 55? 112.512
   D. What PC score must some one obtain to be in the top 5% in the nation? 69.738

31. 3000 freshmen at Powell University took a statistics test. The scores were distributed normally with a mean of 70 and a standard deviation of 5. Use the 68-95-99.7 rule to answer the following questions:
   A. What percentage of scores are between scores 65 and 75? 68%
   B. What percentage of scores are between scores 60 and 70? 47.5%
   C. What percentage of scores are between scores 60 and 85? 97.35%

D. What percentage of scores is less than a score of 55? .15%
E. What percentage of scores is greater than a score of 80? 2.5%
F. Approximately how many biology students scored between 60 and 70? 2850
G. Approximately how many biology students scored between 55 and 60? 70.5

32. Scores for Ms. Powell's calculus class are given below.

90	90	95	100	80	80	75	80	70	60	95	100	100
100	75	80	90	90	90	70	70	80	85	90	90	85

A. Find the mean. 85
B. Find the median. 87.5
C. Find the mode. 90
D. Find the standard deviation. Sample: 10.770 pop: 10.561
E. How many of the scores lie within one standard deviation of the mean? 18
F. How many of the scores lie within two standard deviations of the mean? 25

33. Below are survival times (in days) of 13 guinea pigs that were injected with a bacterial infection in a medical study:

91 83 84 79 91 93 95 97 97 111 101 105 98

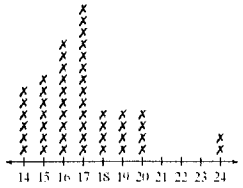
stem	leaf
7	9
8	3 4
9	1 1 3 5 7 7 8
10	1 5
11	1

A. Draw a stem-and-leaf plot.
B. Calculate the standard deviation. 8.936
C. Determine if there are any outliers in the data. none

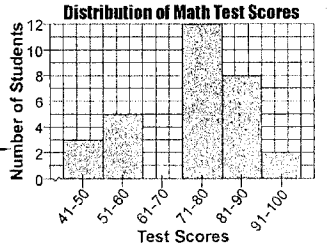
34. Tucker is 60 inches tall. The average height for his age and gender is 68 with a standard deviation of 10 inches. Find Tucker's z-score and percentile using a z-score table. What percent of his peers are shorter than him? 21.19% What percent are taller? 78.81% z = 21.19

35. Describe the population and the sample: A study of high school students at HSHS was conducted to determine on average what percent carry a calculator. Data was collected from the 572 students from the freshman class who carried calculators. pop. sample

36. Use the dotplot to find:
A. the mean 16.92
B. the median 17
C. the mode 17
D. how many people were surveyed 50 people



37. Use the histogram representing the distribution of test scores to answer the following questions:
A. how many students took the test? 30
B. how many passed (scored at 70 or above)? 22
C. what is the difference between the number of scores in the 51-60 range to the number of scores in the 81-90 range? 3



38. A die was rolled 30 times.

6, 5, 4, 4, 5, 6, 1, 2, 1, 6, 4, 3, 3, 3, 4, 2, 2, 5, 6, 4, 1, 2, 4, 3, 5, 5, 3, 3, 4, 2

A. Construct a frequency distribution table for the data. > see solutions
B. Construct a histogram of the data (be sure to label the axes). > see solutions
C. Find the mean. 3.6
D. Find the median. 4
E. Find the mode. 4
F. Find the midrange. 3.5

39. Describe the skewness of each data display.

A.

Stem	Leaf
6	6
7	0 5 7 8
8	1 1 3 4 4 6 8 8 9
9	0 2 9
10	0

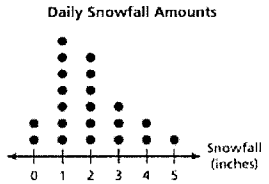
Symmetric

B.

Stem	Leaf
0	1 2 4 5 6 8 9
1	0 1 1 5 7
2	2 5
3	6

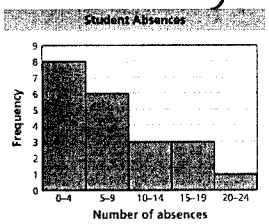
Skewed right

C.



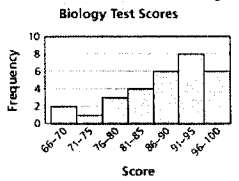
Skewed right

D.



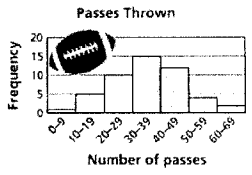
Skewed right

E.



Skewed left

F.



Symmetric