

Practice: The Natural Log Function & Integration

Evaluate.

1)  $\int \frac{5}{x} dx$

$5 \ln|x| + C$

2)  $\int \frac{1}{x-5} dx$

$\ln|x-5| + C$

3)  $\int \frac{x^2-4}{x} dx$

$\frac{1}{2}x^2 - 4 \ln|x| + C$

4)  $\int \frac{x^2-3x+2}{x+1} dx$

$\frac{1}{2}x^2 - 4x + 6 \ln|x+1| + C$

5)  $\int \frac{x^4+x-4}{x^2+2} dx$

$\frac{1}{3}x^3 - 2x + \frac{1}{2} \ln(x^2+2) + C$

6)  $\int \frac{1}{\sqrt{x+1}} dx$

$2\sqrt{x+1} + C$

7)  $\int \frac{1}{\sqrt{2x+1}} dx$

$\sqrt{2x+1} + C$

8)  $\int \frac{\cos \theta}{\sin \theta} d\theta$

$\ln|\sin \theta| + C$

9)  $\int \frac{\cos t}{1+\sin t} dt$

$\ln|1+\sin t| + C$

10) Find  $F'(x)$ .

a)  $F(x) = \int_1^x \frac{1}{t} dt$

$\frac{1}{x}$

b)  $F(x) = \int_1^{3x} \frac{1}{t} dt$

$\frac{1}{x}$

11) Solve the differential equation  $\frac{dy}{dx} = \frac{3}{2-x}$  which passes through the point  $(1, 0)$ .

$y = -3 \ln|2-x|$

12) Find the average value of the function  $f(x) = \frac{8}{x^2}$  over the interval  $[2, 4]$ .

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## Natural Log - Integration

$$1) \int \frac{5}{x} dx = 5 \ln|x| + C$$

$$2) \int \frac{1}{x-5} dx$$

$$\begin{aligned} u &= x-5 & \int \frac{du}{u} &= \ln|u| + C = \ln|x-5| + C \\ \frac{du}{dx} &= 1 & du &= dx \end{aligned}$$

$$3) \int \frac{x^2-4}{x} dx = \int \left(x - \frac{4}{x}\right) dx = \frac{1}{2}x^2 - 4 \ln|x| + C$$

$$4) \int \frac{x^2-3x+2}{x+1} dx = \int \left(x-4 + \frac{6}{x+1}\right) dx = \frac{1}{2}x^2 - 4x + 6 \ln|x+1| + C$$

$$\begin{array}{r} -1 \\ \underline{-} \quad 1 \quad -3 \quad 2 \\ \underline{-1} \quad \underline{4} \\ 1 \quad -4 \quad 6 \end{array}$$

$$\begin{aligned} &\int \frac{6}{x+1} dx \\ u &= x+1 & 6 \int \frac{du}{u} &= 6 \ln|u| + C \\ \frac{du}{dx} &= 1 & 6 \ln|x+1| + C \\ du &= dx \end{aligned}$$

$$5) \int \frac{x^4+x-4}{x^2+2} dx = \int \left(x^2 - 2 + \frac{x}{x^2+2}\right) dx = \frac{1}{3}x^3 - 2x + \frac{1}{2} \ln(x^2+2) + C$$

$$\begin{array}{r} x^2 - 2 \\ x^2 + 2 ) \overline{x^4 + 0x^3 + 0x^2 + x - 4} \\ \underline{-} (x^4 \qquad \qquad \qquad + 2x^2) \\ \underline{\underline{-2x^2 + x}} \qquad \qquad \qquad \downarrow \\ - (-2x^2 \qquad \qquad \qquad - 4) \\ \qquad \qquad \qquad x \end{array}$$

$$\begin{aligned} &\int \frac{x}{x^2+2} dx \\ u &= x^2+2 & \frac{1}{2} \int \frac{du}{u} &= \frac{1}{2} \ln|u| + C \\ \frac{du}{dx} &= 2x & \frac{1}{2} \ln|x^2+2| + C \\ \frac{1}{2} du &= x dx \end{aligned}$$

$$6) \int \frac{1}{\sqrt{x+1}} dx$$

$$\begin{aligned} u &= x+1 \\ \frac{du}{dx} &= 1 & du &= dx \end{aligned}$$

$$\begin{aligned} \int \frac{1}{\sqrt{u}} du &= \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C \\ &= 2\sqrt{x+1} + C \end{aligned}$$

$$7) \int \frac{1}{\sqrt{2x+1}} dx$$

$$\begin{aligned} u &= 2x+1 & \frac{1}{2} \int \frac{1}{\sqrt{u}} du &= \frac{1}{2} \int u^{-\frac{1}{2}} du = u^{\frac{1}{2}} + C \\ \frac{du}{dx} &= 2 & &= \sqrt{2x+1} + C \\ \frac{1}{2} du &= dx \end{aligned}$$

$$8) \int \frac{\cos \theta}{\sin \theta} d\theta = \int \cot \theta d\theta = \ln |\sin \theta| + C$$

$$9) \int \frac{\cos t}{1+\sin t} dt$$

$$u = 1 + \sin t$$

$$\frac{du}{dt} = \cos t$$

$$du = \cos t dt$$

$$\int \frac{du}{u} = \ln |u| + C = \ln |1 + \sin t| + C$$

$$10) \quad a) \frac{d}{dx} \int_1^x \frac{1}{t} dt = \frac{1}{x}$$

$$b) \frac{d}{dx} \int_1^{3x} \frac{1}{t} dt = \frac{1}{3x} \cdot 3 = \frac{1}{x}$$

$$11) \int dy = \int \frac{3}{2-x} dx$$

$$y = -3 \ln |2-x| + C$$

$$0 = -3 \ln |2-1| + C$$

$$0 = C$$

$$y = -3 \ln |2-x|$$

$$u = 2-x \quad -3 \int \frac{du}{u} = -3 \ln |u| + C$$

$$\frac{du}{dx} = -1 \quad -du = dx$$

$$12) \frac{1}{4-2} \int_2^4 8x^{-2} dx = \frac{1}{2} \left[ -8x^{-1} + C \Big|_2^4 \right] = \frac{1}{2} \left[ -\frac{8}{4} + C - \left( -\frac{8}{2} + C \right) \right] = \frac{1}{2} [2] = 1$$