Section 1.4 Exercises

1. $(f+g)(x) = 2x - 1 + x^2$; $(f-g)(x) = 2x - 1 - x^2$; $(fg)(x) = (2x-1)(x^2) = 2x^3 - x^2$.
   There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.

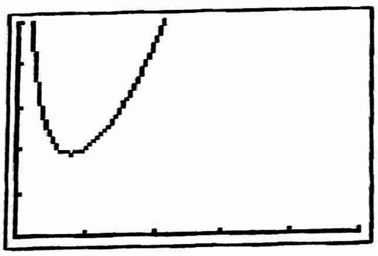
2. $(f+g)(x) = (x-1)^2 + 3 - x = x^2 - 2x + 1 + 3 - x = x^2 - 3x + 4$;
   $(f-g)(x) = (x-1)^2 - 3 + x = x^2 - 2x + 1 - 3 + x = x^2 - x - 2$;
   $$(fg)(x) = (x-1)^2(3-x) = (x^2 - 2x + 1)(3-x)$$
   $$= 3x^2 - x^3 - 6x + 2x^2 + 3 - x$$
   $$= -x^3 + 5x^2 - 7x + 3.$$
   There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.

3. $(f+g)(x) = \sqrt{x} + \sin x$; $(f-g)(x) = \sqrt{x} - \sin x$; $(fg)(x) = \sqrt{x}\sin x$.
   Domain in each case is $[0, \infty)$. For $\sqrt{x}$, $x \ge 0$. For $\sin x$, $-\infty < x < \infty$.

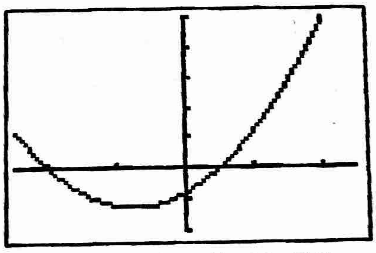
4. $(f+g)(x) = \sqrt{x+5} + |x+3|$;
   $(f-g)(x) = \sqrt{x+5} - |x+3|$;
   $(fg)(x) = |x+3|\sqrt{x+5}$;
   All three expressions contain $\sqrt{x+5}$, so $x + 5 \ge 0$ and $x \ge -5$; all three domains are $[-5, \infty)$. For $|x+3|$, $-\infty < x < \infty$.

5. $(f/g)(x) = \dfrac{\sqrt{x+3}}{x^2}$; $x + 3 \ge 0$ and $x \ne 0$, so the domain is $[-3, 0) \cup (0, \infty)$.
   $(g/f)(x) = \dfrac{x^2}{\sqrt{x+3}}$; $x + 3 > 0$, so the domain is $(-3, \infty)$.

6. $(f/g)(x) = \dfrac{\sqrt{x-2}}{\sqrt{x+4}} = \sqrt{\dfrac{x-2}{x+4}}$; $x - 2 \ge 0$ and $x + 4 > 0$, so $x \ge 2$ and $x > -4$; the domain is $[2, \infty)$.
   $(g/f)(x) = \dfrac{\sqrt{x+4}}{\sqrt{x-2}} = \sqrt{\dfrac{x+4}{x-2}}$; $x + 4 \ge 0$ and $x - 2 > 0$, so $x \ge -4$ and $x > 2$; the domain is $(2, \infty)$.

7. 

   $[0, 5]$ by $[0, 5]$

8. 

   $[-5, 5]$ by $[-10, 25]$

9. $(f \circ g)(3) = f(g(3)) = f(4) = 5$; $(g \circ f)(-2) = g(f(-2)) = g(-7) = -6$

10. $(f \circ g)(3) = f(g(3)) = f(3) = 8$; $(g \circ f)(-2) = g(f(-2)) = g(3) = 3$

11. $(f \circ g)(x) = f(g(x)) = 3(x-1) + 2 = 3x - 3 + 2 = 3x - 1$. Since both f and g have domain $(-\infty, \infty)$, the domain of $(f \circ g)$ is $(-\infty, \infty)$.
    $(g \circ f)(x) = g(f(x)) = (3x + 2) - 1 = 3x + 1$; again, the domain is $(-\infty, \infty)$.

12. $(f \circ g)(x) = f(g(x)) = \left(\frac{1}{x-1}\right)^2 - 1$

$= \frac{1}{(x-1)^2} - 1$. The domain of g is $x \neq 1$, while the domain of f is $(-\infty, \infty)$, so the domain of $(f \circ g)$ is $x \neq 1$, or $(-\infty, 1) \cup (1, \infty)$.

$(g \circ f)(x) = g(f(x)) = \frac{1}{(x^2-1)-1} = \frac{1}{x^2-2}$.

The domain of f is $(-\infty, \infty)$, while the domain of g is $(-\infty, 1) \cup (1, \infty)$, so $g \circ f$ requires that $f(x) \neq 1$. This means $x^2 - 1 \neq 1$, or $x^2 \neq 2$, so the domain of $g \circ f$ is $x \neq \pm\sqrt{2}$, or $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

13. $(f \circ g)(x) = f(g(x)) = (\sqrt{x+1})^2 - 2$
$= x + 1 - 2 = x - 1$. The domain of g is $x \geq -1$, while the domain of f is $(-\infty, \infty)$, so the domain of $(f \circ g)$ is $x \geq -1$, or $[-1, \infty)$.

$(g \circ f)(x) = g(f(x)) = \sqrt{(x^2-2)+1} = \sqrt{x^2-1}$.

The domain of f is $(-\infty, \infty)$, while the domain of g is $[-1, \infty)$, so $g \circ f$ requires that $f(x) \geq -1$. This means $x^2 - 2 \geq -1$, or $x^2 \geq 1$, which means $x \leq -1$ or $x \geq 1$. Therefore the domain of $g \circ f$ is $(-\infty, -1] \cup [1, \infty)$.

14. $(f \circ g)(x) = f(g(x)) = \frac{1}{\sqrt{x}-1}$. The domain of g is $x \geq 0$, while the domain of f is $(-\infty, 1) \cup (1, \infty)$, so $f \circ g$ requires that $x \geq 0$ and $g(x) \neq 1$, or $x \geq 0$, and $x \neq 1$. The domain of $f \circ g$ is $[0, 1) \cup (1, \infty)$.

$(g \circ f)(x) = g(f(x)) = \sqrt{\frac{1}{x-1}} = \frac{1}{\sqrt{x-1}}$. The domain of f is $x \neq 1$, while the domain of g is $[0, \infty)$, so $g \circ f$ requires that $x \neq 1$ and $f(x) \geq 0$, or $x \neq 1$ and $\frac{1}{x-1} \geq 0$. The latter occurs if $x - 1 > 0$, so the domain of $g \circ f$ is $(1, \infty)$.

15. One possibility: $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5x$

16. One possibility: $f(x) = (x+1)^2$ and $g(x) = x^3$

17. One possibility: $f(x) = |x|$ and $g(x) = 3x - 2$

18. One possibility: $f(x) = 1/x$ and $g(x) = x^3 - 5x + 3$

19. One possibility: $f(x) = x^5 - 2$ and $g(x) = x - 3$

20. One possibility: $f(x) = e^x$ and $g(x) = \sin x$

27. $y^2 = 25 - x^2$, $y = \sqrt{25 - x^2}$ and $y = -\sqrt{25 - x^2}$

28. $y^2 = 25 - x$, $y = \sqrt{25 - x}$ and $y = -\sqrt{25 - x}$

29. $y^2 = x^2 - 25$, $y = \sqrt{x^2 - 25}$ and $y = -\sqrt{x^2 - 25}$

30. $y^2 = 3x^2 - 25$, $y = \sqrt{3x^2 - 25}$ and $y = -\sqrt{3x^2 - 25}$