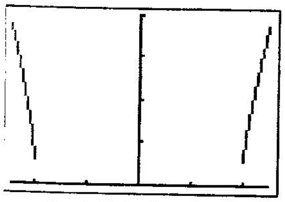
We need
$$x^4 - 16x^2 \geq 0$$
$$x^2(x^2 - 16) \geq 0$$
$$x^2 = 0 \quad \text{or} \quad x^2 - 16 \geq 0$$
$$x^2 \geq 16$$
$$x = 0 \quad \text{or} \quad x \geq 4, \quad x \leq -4$$
Domain: $(-\infty, -4] \cup \{0\} \cup [4, \infty)$

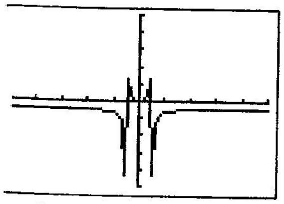


$[-5, 5]$ by $[0, 16]$

$f(x) = 10 - x^2$ can take on any negative value. Because x^2 is nonnegative, $f(x)$ cannot be greater than 10. The range is $(-\infty, 10]$.

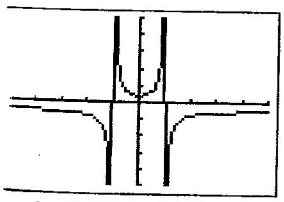
$g(x) = 5 + \sqrt{4 - x}$ can take on any value ≥ 5, but because $\sqrt{4 - x}$ is nonnegative, $g(x)$ cannot be less than 5. The range is $[5, \infty)$.

The range of a function is most simply found by graphing it. As our graph shows, the range of $f(x)$ is $(-\infty, -1) \cup [0, \infty)$.



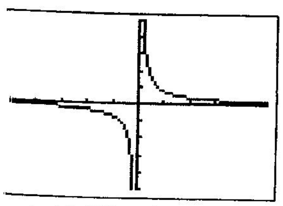
$[-10, 10]$ by $[-10, 10]$

As our graph illustrates, the range of $g(x)$ is $(-\infty, -1) \cup [\frac{3}{4}, \infty)$.



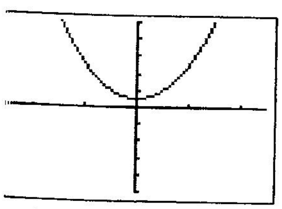
$[-10, 10]$ by $[-10, 10]$

Yes, non-removable



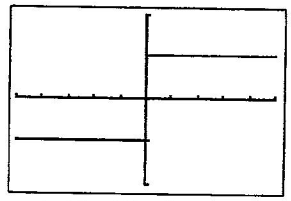
$[-10, 10]$ by $[-10, 10]$

Yes, removable



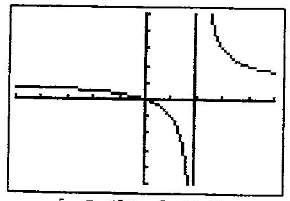
$[-5, 5]$ by $[-10, 10]$

23. Yes, non-removable



$[-10, 10]$ by $[-2, 2]$

24. Yes, non-removable



$[-5, 5]$ by $[-5, 5]$

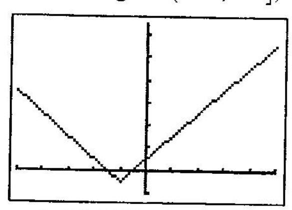
25. Local maxima at $(-1, 4)$ and $(5, 5)$, local minimum at $(2, 2)$. The function increases on $(-\infty, -1]$, decreases on $[-1, 2]$, increases on $[2, 5]$, and decreases on $[5, \infty)$.

26. Local minimum at $(1, 2)$, $(3, 3)$ is neither, and $(5, 7)$ is a local maximum. The function decreases on $(-\infty, 1]$, increases on $[1, 5]$, and decreases on $[5, \infty)$.

27. $(-1, 3)$ and $(3, 3)$ are neither. $(1, 5)$ is a local maximum, and $(5, 1)$ is a local minimum. The function increases on $(-\infty, 1]$, decreases on $[1, 5]$, and increases on $[5, \infty)$.

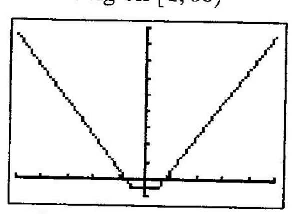
28. $(-1, 1)$ and $(3, 1)$ are local minima, while $(1, 6)$ and $(5, 4)$ are local maxima. The function decreases on $(-\infty, -1]$, increases on $[-1, 1]$, decreases on $(1, 3]$, increases on $[3, 5]$, and decreases on $[5, \infty)$.

29. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



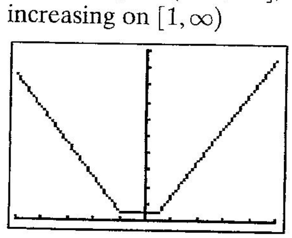
$[-10, 10]$ by $[-2, 18]$

30. Decreasing on $(-\infty, -1]$; constant on $[-1, 1]$; increasing on $[1, \infty)$



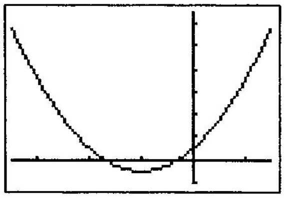
$[-10, 10]$ by $[-2, 18]$

31. Decreasing on $(-\infty, -2]$; constant on $[-2, 1]$; increasing on $[1, \infty)$



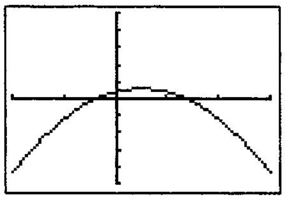
$[-10, 10]$ by $[0, 20]$

32. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



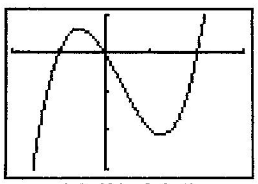
$[-7, 3]$ by $[-2, 13]$

33. Increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



$[-4, 6]$ by $[-25, 25]$

34. Increasing on $(-\infty, -0.5]$; decreasing on $[-0.5, 1.2]$, increasing on $[1.2, \infty)$. The middle values are approximate—they are actually at about -0.549 and 1.215. The values given are what might be observed on the decimal window.



$[-2, 3]$ by $[-3, 1]$

35. Constant functions are always bounded.

36.
$$x^2 > 0$$
$$-x^2 < 0$$
$$2 - x^2 < 2$$
y is bounded above by $y = 2$.

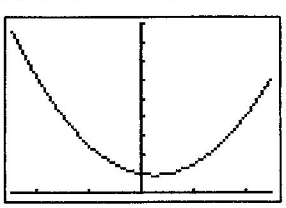
37. $2^x > 0$ for all x, so y is bounded below by $y = 0$.

38. $2^{-x} = \dfrac{1}{2^x} > 0$ for all x, so y is bounded below by $y = 0$.

39. Since $y = \sqrt{1 - x^2}$ is always positive, we know that $y \geq 0$ for all x. We must also check for an upper bound:
$$x^2 > 0$$
$$-x^2 < 0$$
$$1 - x^2 < 1$$
$$\sqrt{1 - x^2} < \sqrt{1}$$
$$\sqrt{1 - x^2} < 1$$
Thus, y is bounded.

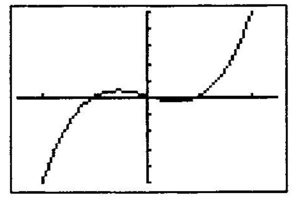
40. There are no restrictions on either x or x^3, so y is not bounded.

41. f has a local minimum when $x = 0.5$, where $y = 3.75$. It has no maximum.



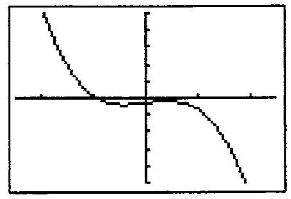
$[-5, 5]$ by $[0, 36]$

42. Local maximum: $y \approx 4.08$ at $x \approx -1.15$.
Local minimum: $y \approx -2.08$ at $x \approx 1.15$.



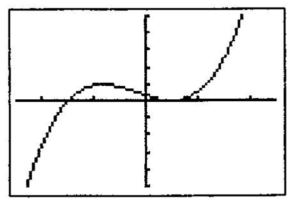
$[-5, 5]$ by $[-50, 50]$

43. Local minimum: $y \approx -4.09$ at $x \approx -0.82$.
Local maximum: $y \approx -1.91$ at $x \approx 0.82$.



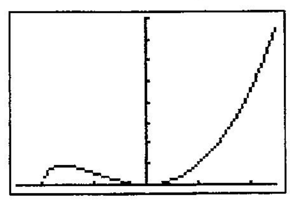
$[-5, 5]$ by $[-50, 50]$

44. Local maximum: $y \approx 9.48$ at $x \approx -1.67$.
Local minimum: $y = 0$ when $x = 1$.



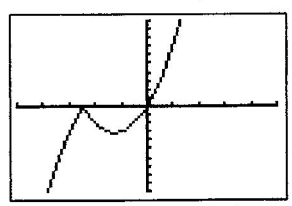
$[-5, 5]$ by $[-50, 50]$

45. Local maximum: $y \approx 9.16$ at $x \approx -3.20$.
Local minima: $y = 0$ at $x = 0$ and $y = 0$ at $x = -4$.



$[-5, 5]$ by $[0, 80]$

46. Local maximum: $y = 0$ at $x = -2.5$.
Local minimum: $y \approx -3.13$ at $x = -1.25$.



$[-5, 5]$ by $[-10, 10]$

47. Even: $f(-x) = 2(-x)^4 = 2x^4 = f(x)$

48. Odd: $g(-x) = (-x)^3 = -x^3 = -g(x)$

49. Even: $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

50. Even: $g(-x) = \dfrac{3}{1 + (-x)^2} = \dfrac{3}{1 + x^2} = g(x)$

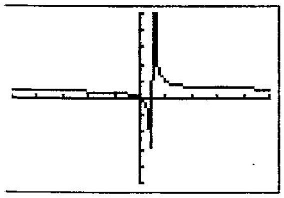
51. Neither: $f(-x) = -(-x)^2 + 0.03(-x) + 5 = -x^2 - 0.03x + 5$, which is neither $f(x)$ nor $-f(x)$.

52. Neither: $f(-x) = (-x)^3 + 0.04(-x)^2 + 3 = -x^3 + 0.04x^2 + 3$, which is neither $f(x)$ nor $-f(x)$.

53. Odd: $g(-x) = 2(-x)^3 - 3(-x) = -2x^3 + 3x = -g(x)$

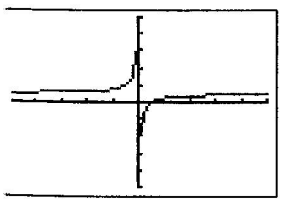
54. Odd: $h(-x) = \frac{1}{-x} = -\frac{1}{x} = -h(x)$

55. The quotient $\frac{x}{x-1}$ is undefined at $x = 1$, indicating that $x = 1$ is a vertical asymptote. Similarly, $\lim_{x\to\infty} \frac{x}{x-1} = 1$, indicating a horizontal asymptote at $y = 1$. The graph confirms these.



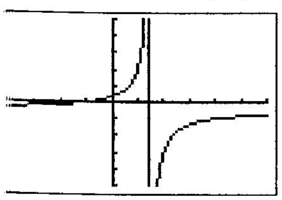
[−10, 10] by [−10, 10]

56. The quotient $\frac{x-1}{x}$ is undefined at $x = 0$, indicating a possible vertical asymptote at $x = 0$. Similarly, $\lim_{x\to\infty} \frac{x-1}{x} = 1$, indicating a possible horizontal asymptote at $y = 1$. The graph confirms those asymptotes.



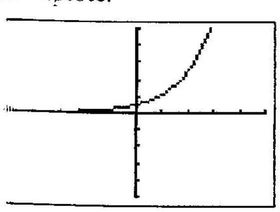
[−10, 10] by [−10, 10]

57. The quotient $\frac{x+2}{3-x}$ is undefined at $x = 3$, indicating a possible vertical asymptote at $x = 3$. Similarly, $\lim_{x\to\infty} \frac{x+2}{3-x} = -1$, indicating a possible horizontal asymptote at $y = -1$. The graph confirms these asymptotes.



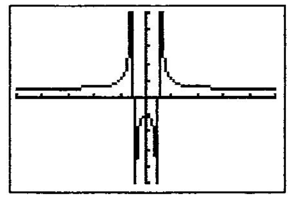
[−8, 12] by [−10, 10]

58. Since $g(x)$ is continuous over $-\infty < x < \infty$, we do not expect a vertical asymptote. However, $\lim_{x\to-\infty} 1.5^x = \lim_{x\to\infty} 1.5^{-x} = \lim_{x\to\infty} \frac{1}{1.5^x} = 0$, so we expect a horizontal asymptote $y = 0$. The graph confirms this asymptote.



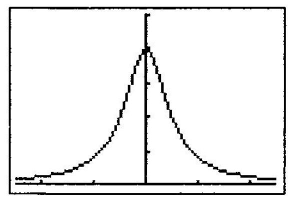
[−10, 10] by [−10, 10]

59. The quotient $\frac{x^2+2}{x^2-1}$ is undefined at $x = 1$ and $x = -1$. So we expect two vertical asymptotes. Similarly, the $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = 1$, so we expect a horizontal asymptote at $y = 1$. The graph confirms theses asymptotes.



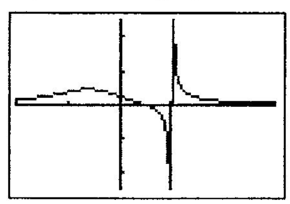
[−10, 10] by [−10, 10]

60. We note that $x^2 + 1 > 0$ for $-\infty < x < \infty$, so we do not expect a vertical asymptote. However, $\lim_{x\to\infty} \frac{4}{x^2+1} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms this.



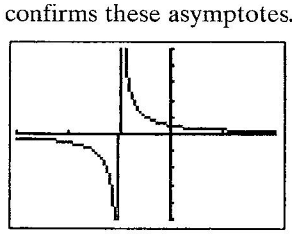
[−5, 5] by [0, 5]

61. The quotient $\frac{4x-4}{x^3-8}$ does not exist at $x = 2$, so we expect a vertical asymptote there. Similarly, $\lim_{x\to\infty} \frac{4x-4}{x^3+8} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



[−4, 6] by [−5, 5]

62. The quotient $\frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x-2)(x+2)} = \frac{2}{x+2}$. Since $x = 2$ is a removable discontinuity, we expect a vertical asymptote at only $x = -2$. Similarly, $\lim_{x\to\infty} \frac{2}{x-2} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



[−6, 4] by [−10, 10]

63. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x+2}{2x+1}$ behaves much like $\frac{x}{2x} = \frac{1}{2}$, so there is a horizontal asymptote at $y = \frac{1}{2}$. The graph matching this description is (b).

64. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x^2+2}{2x+1}$ behaves much like $\frac{x^2}{2x} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (c).

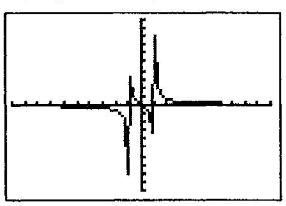
65. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x+2}{2x^2+1}$ behaves much like $\frac{x}{2x^2} = \frac{1}{2x}$, which for large x is close to zero. So there is a horizontal asymptote at $y = 0$. The graph matching this description is (a).

66. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x^3+2}{2x^2+1}$ behaves much like $\frac{x^3}{2x^2} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (d).

67. (a) Since, $\lim_{x\to\infty} \frac{x}{x^2-1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation

$$\frac{x}{x^2-1} = 0$$
$$x = 0\cdot(x^2-1)$$
$$x = 0$$

The graph confirms that $f(x)$ crosses the horizontal asymptote at $(0, 0)$.

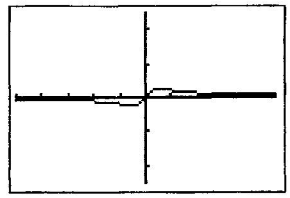


[−10, 10] by [−10, 10]

(b) Since $\lim_{x\to\infty} \frac{x}{x^2+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation:

$$\frac{x}{x^2+1} = 0$$
$$x = 0\cdot(x^2+1)$$
$$x = 0$$

The graph confirms that $g(x)$ crosses the horizontal asymptote at $(0, 0)$.

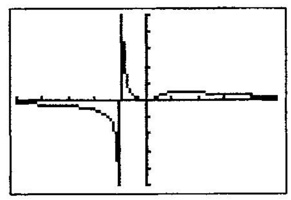


[−10, 10] by [−5, 5]

(c) Since $\lim_{x\to\infty} \frac{x^2}{x^3+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where $h(x)$ crosses $y = 0$, we solve the equation

$$\frac{x^2}{x^3+1} = 0$$
$$x^2 = 0\cdot(x^3+1)$$
$$x^2 = 0$$
$$x = 0$$

The graph confirms that $h(x)$ crosses the horizontal asymptote at $(0, 0)$.

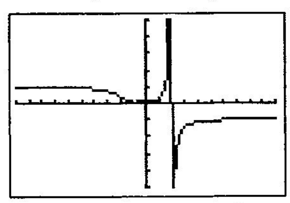


[−5, 5] by [−5, 5]

68. (a) To find horizontal asymptotes, we check limits, at $x \to \infty$ and $x \to -\infty$. We also know that our numerator $|x^3+1|$, is positive for all x, and that our denominator, $8 - x^3$, is positive for $x < 2$ and negative for $x > 2$. Considering these two statements, we find

$$\lim_{x\to\infty} \frac{|x^3+1|}{8-x^3} = -1 \text{ and } \lim_{x\to-\infty} \frac{|x^3+1|}{8-x^3} = 1.$$

The graph confirms that we have horizontal asymptotes at $y = 1$ and $y = -1$.



[−10, 10] by [−5, 5]

(b) Again, we see that our numerator, $|x-1|$, is positive for all x. As a result, $g(x)$ can be negative only when $x^2 - 4 < 0$, and $g(x)$ can be positive only when $x^2 - 4 > 0$. This means that $g(x)$ can be negative only when $-2 < x < 2$; if $x < -2$ or $x > 2$, $g(x)$ will be positive. As a result, we know that

$$\lim_{x\to\infty} \frac{|x-1|}{x^2-4} = \lim_{x\to-\infty} \frac{|x-1|}{x^2-4} = 0,$$ giving just one horizontal asymptote at $y = 0$. Our graph confirms this asymptote.