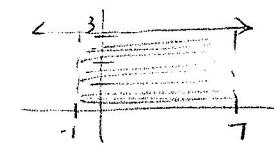


AP Calculus

Definite Integration Homework

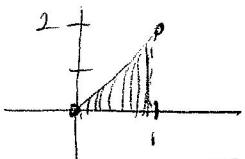
1. Evaluate each definite integral.

$$\int_{-1}^7 3 \, dx$$



$$\text{rect } A = 8 \cdot 3 = \boxed{24}$$

$$\int_0^1 2x \, dx$$



$$\text{triangle } A = \frac{1}{2}(1)(2) = \boxed{1}$$

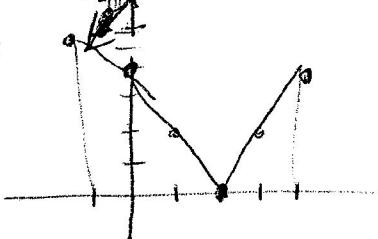
$$\int_{-1}^0 (x - 2) \, dx$$



$$\text{trap. } \frac{1}{2}(1)(3+2) = \frac{5}{2}$$

$$\boxed{-\frac{5}{2}}$$

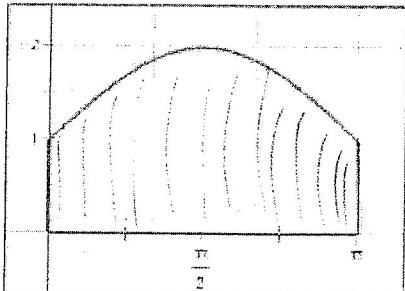
$$\int_{-1}^4 |2x - 4| \, dx$$



2 triangles

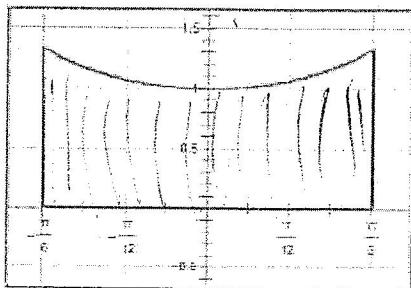
$$\frac{1}{2}(3)(6) + \frac{1}{2}(2)(4) = 9 + 4 = \boxed{13}$$

2. Write an integral to represent the shaded area.



$$\int_0^{\pi} (1 + \sin x) \, dx$$

$$y = 1 + \sin x$$



$$y = \sec^2 x$$

$$\int_{-\pi/6}^{\pi/6} (\sec^2 x) \, dx$$

3. Suppose that f and g are continuous functions with the below given information. Use the properties of definite integrals to evaluate each expression:

$$\int_1^2 f(x) dx = -4$$

$$\int_1^5 f(x) dx = 6$$

$$\int_1^5 g(x) dx = 8$$

$$\int_2^2 g(x) dx = \boxed{0}$$

$$\int_2^5 f(x) dx = \int_1^5 f(x) dx - \int_1^2 f(x) dx \\ = 6 - (-4)$$

$$\int_5^1 g(x) dx = - \int_1^5 g(x) dx = \boxed{-8}$$

$$\int_1^5 [f(x) + g(x)] dx = \boxed{10} \\ = 6 + 8 = \boxed{14}$$

$$\int_1^2 3f(x) dx = 3 \int_1^2 f(x) dx = 3(-4) \\ = \boxed{-12}$$

$$\int_1^5 [4f(x) - g(x)] dx = 4(6) - 8 \\ = 24 - 8 \\ = \boxed{16}$$

4. Suppose that f and g are continuous functions with the below given information. Use the properties of definite integrals to evaluate each expression:

$$\int_1^9 f(x) dx = -1$$

$$\int_7^9 f(x) dx = 5$$

$$\int_7^9 h(x) dx = 4$$

$$\int_9^1 f(x) dx = -(-1) = \boxed{1}$$

$$\int_1^9 -2f(x) dx = -2(-1) = \boxed{2}$$

$$\int_1^7 f(x) dx = \int_1^9 f(x) dx - \int_7^9 f(x) dx \quad \int_7^9 [f(x) + h(x)] dx = 4 + 5 = \boxed{9}$$

$$= -1 - 5 \\ = \boxed{-6}$$

$$\int_9^7 [h(x) - f(x)] dx \\ = - \int_7^9 (h(x) - f(x)) dx \\ = - [\boxed{4 - 5}] \\ = \boxed{1}$$

$$\int_7^9 [2f(x) - 3h(x)] dx = 2(5) - 3(4)$$

$$= 10 - 12 \\ = \boxed{-2}$$

AP Calculus AB

Homework – Antiderivatives and Indefinite Integration

Find the Indefinite Integral and check the result by differentiation.

1. $\int (x + 7) dx$

$\frac{1}{2}x^2 + 7x + C$

2. $\int (13 - x) dx$

$13x - \frac{1}{2}x^2 + C$

3. $\int (2x - 3x^2) dx$

$x^2 - x^3 + C$

4. $\int (8x^3 - 9x^2 + 4) dx$

$2x^4 - 3x^3 + 4x + C$

5. $\int (x^5 + 1) dx$

$\frac{1}{6}x^6 + x + C$

6. $\int (x^3 - 10x - 3) dx$

$\frac{1}{4}x^4 - 5x^2 - 3x + C$

7. $\int (x^{\frac{3}{2}} + 2x + 1) dx$

$\frac{2}{5}x^{\frac{5}{2}} + x^2 + x + C$

8. $\int (\sqrt{x} + \frac{1}{2\sqrt{x}}) dx$

$= \int (x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}) dx$
 $\frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$

9. $\int \sqrt[3]{x^2} dx$
 $= \int (x^{\frac{2}{3}})^3 dx$
 $= \frac{3}{5}x^{\frac{5}{3}} + C$

10. $\int (\sqrt[4]{x^3} + 1) dx$

$= \int (x^{\frac{3}{4}} + 1) dx$

$\frac{4}{7}x^{\frac{7}{4}} + x + C$

11. $\int \frac{1}{x^6} dx$

$= \int x^{-6} dx$

$-\frac{1}{5}x^{-5} + C$

$= -\frac{1}{5x^5} + C$

12. $\int \frac{x+6}{\sqrt{x}} dx$

$= \int (x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}) dx$

$\frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} + C$

13. $\int \frac{x^2+2x-3}{x^4} dx$

$= \int (x^{-2} + 2x^{-3} - 3x^{-4}) dx$

$= -x^{-1} - x^{-2} + x^{-3} + C$

14. $\int (x+1)(3x-2) dx$

$= \int (3x^2 + x - 2) dx$

$x^3 + \frac{1}{2}x^2 - 2x + C$

15. $\int x^2 \sqrt{x} dx = \int (x^2 \cdot x^{\frac{1}{2}}) dx$

$= \int x^{\frac{5}{2}} dx$

$\frac{2}{7}x^{\frac{7}{2}} + C$

12. $\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

$$6. \int dx$$

$$x + C$$

$$17. \int (x^2 - \cos x) dx$$

$$\frac{1}{3}x^3 - \sin x + C$$

$$18. \int (5 \cos x + 4 \sin x) dx$$

$$5 \sin x - 4 \cos x + C$$

$$19. \int (1 - \csc x \cot x) dx$$

$$x + \csc x + C$$

$$20. \int (\sec^2 x - \sin x) dx$$

$$\tan x + \cos x + C$$

$$\begin{aligned} 21. \int \frac{\cos x}{1 - \cos^2 x} dx &= \int \frac{\cos x}{\sin^2 x} dx \\ &= \int \left(\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \right) dx \\ &= \int (\cot x \csc x) dx \\ &\quad - \csc x + C \end{aligned}$$

Find f(x) given the initial condition:

$$22. f'(x) = 8x^3 + 5, f(1) = -4$$

$$f(x) = \int (8x^3 + 5) dx = 2x^4 + 5x + C$$

$$-4 = 2(1)^4 + 5(1) + C$$

$$C = -11$$

$$f(x) = 2x^4 + 5x - 11$$

$$24. f'(x) = 3x^2, f(1) = 6$$

$$f(x) = x^3 + C$$

$$6 = (1)^3 + C$$

$$S = C$$

$$f(x) = x^3 + 5$$

$$26. f''(x) = x^2, f'(0) = 8, f(0) = 4$$

$$f'(x) = \frac{1}{3}x^3 + C$$

$$8 = \frac{1}{3}(0)^3 + C$$

$$8 = C$$

$$f'(x) = \frac{1}{3}x^3 + 8$$

$$f(x) = \frac{1}{12}x^4 + 8x + C$$

$$4 = \frac{1}{12}(0)^4 + 8(0) + C$$

$$4 = C$$

$$f(x) = \frac{1}{12}x^4 + 8x + 4$$

$$23. f'(x) = 6x, f(0) = 8$$

$$f(x) = \int 6x dx = 3x^2 + C$$

$$8 = 3(0)^2 + C$$

$$C = 8$$

$$f(x) = 3x^2 + 8$$

$$25. f''(x) = 2, f'(2) = 5, f(2) = 10$$

$$f'(x) = 2x + C$$

$$5 = 2(2) + C$$

$$C = 1 \Rightarrow f'(x) = 2x + 1$$

$$f(x) = x^2 + x + C$$

$$10 = (2)^2 + 2 + C$$

$$C = 4$$

$$f(x) = x^2 + x + 4$$

$$27. f''(x) = \sin x, f'(0) = 1, f(0) = 6$$

$$f'(x) = -\cos x + C$$

$$1 = -\cos(0) + C$$

$$2 = C$$

$$f'(x) = -\cos x + 2$$

$$f(x) = -\sin x + 2x + C$$

$$6 = -\sin(0) + 2(0) + C$$

$$6 = C$$

$$f(x) = -\sin x + 2x + 6$$