NAME _____ DATE _____ PERIOD _____

14-6

Skills Practice

Double-Angle and Half-Angle Formulas

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

1. $\cos \theta = \frac{7}{25}$, $0° < \theta < 90°$

$\frac{336}{625}$, $-\frac{527}{625}$, $\frac{3}{5}$, $\frac{4}{5}$

2. $\sin \theta = -\frac{4}{5}$, $180° < \theta < 270°$

3. $\sin \theta = \frac{40}{41}$, $90° < \theta < 180°$

4. $\cos \theta = \frac{3}{7}$, $270° < \theta < 360°$

$-\frac{12\sqrt{10}}{49}$, $-\frac{31}{49}$, $\frac{\sqrt{14}}{7}$, $-\frac{\sqrt{35}}{7}$

5. $\cos \theta = -\frac{3}{5}$, $90° < \theta < 180°$

6. $\sin \theta = \frac{5}{13}$, $0° < \theta < 90°$

Find the exact value of each expression by using the half-angle formulas.

7. $\cos 22\frac{1}{2}°$

8. $\sin 165°$

9. $\cos 105°$

10. $\sin \frac{\pi}{8}$

11. $\sin \frac{15\pi}{8}$ $-\frac{\sqrt{2-\sqrt{2}}}{2}$

12. $\cos 75°$

Verify that each of the following is an identity.

13. $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

14. $\tan \theta + \cot \theta = 2 \csc 2\theta$

$$\frac{2 \cdot \frac{\sin\theta}{\cos\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} = \frac{2\sin\theta}{\cos\theta} \cdot \underbrace{\frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta}}_{1} = 2\sin\theta\cos\theta = \sin(2\theta)$$

NAME _____ DATE _____ PERIOD _____

14-6

Practice

Double-Angle and Half-Angle Formulas

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

1. $\cos \theta = \frac{5}{13}$, $0° < \theta < 90°$

2. $\sin \theta = \frac{8}{17}$, $90° < \theta < 180°$

$-\frac{240}{289}$, $\frac{161}{289}$, $\frac{4\sqrt{17}}{17}$, $\frac{\sqrt{17}}{17}$

3. $\cos \theta = \frac{1}{4}$, $270° < \theta < 360°$

4. $\sin \theta = -\frac{2}{3}$, $180° < \theta < 270°$

Find the exact value of each expression by using the half-angle formulas.

5. tan 105°

$-2-\sqrt{3}$

6. tan 15°

7. cos 67.5°

8. $\sin\left(-\frac{\pi}{8}\right)$

Verify that each of the following is an identity.

9. $\sin^2 \frac{\theta}{2} = \frac{\tan \theta - \sin \theta}{2 \tan \theta}$

$\sin^2\frac{\theta}{2} = \left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = \left(\frac{1-\cos\theta}{2}\right)\cdot\left(\frac{\tan\theta}{\tan\theta}\right) = \frac{\tan\theta - \cos\theta\cdot\frac{\sin\theta}{\cos\theta}}{2\tan\theta} = \frac{\tan\theta - \sin\theta}{2\tan\theta}$

10. $\sin 4\theta = 4 \cos 2\theta \sin \theta \cos \theta$

$\sin 4\theta = \sin(2\cdot 2\theta) = 2\sin 2\theta\cos 2\theta = 2\cdot(2\sin\theta\cos\theta)\cos 2\theta$
$= 4\cdot\sin\theta\cos\theta\cdot\cos 2\theta$

11. AERIAL PHOTOGRAPHY In aerial photography, there is a reduction in film exposure for any point X not directly below the camera. The reduction E_θ is given by $E_\theta = E_0 \cos^4 \theta$, where θ is the angle between the perpendicular line from the camera to the ground and the line from the camera to point X, and E_0 is the exposure for the point directly below the camera. Using the identity $2 \sin^2 \theta = 1 - \cos 2\theta$, verify that $E_0 \cos^4 \theta = E_0\left(\frac{1}{2} + \frac{\cos 2\theta}{2}\right)^2$.

12. IMAGING A scanner takes thermal images from altitudes of 300 to 12,000 meters. The width W of the swath covered by the image is given by $W = 2H' \tan \theta$, where H' is the height and θ is half the scanner's field of view. Verify that $\frac{2H' \sin 2\theta}{1 + \cos 2\theta} = 2H' \tan \theta$.