Note that in the rectangular system, there is only one way to label a point. In the polar system, there are several ways to label a point, actually an infinite number of ways.

Example 1) For each polar point, label it in two other ways:

a. $(4,60°)$
$\boxed{(-4,240°),(4,-300°)}$

b. $(-5,315°)$
$\boxed{(5,135°),(5,-45°)}$

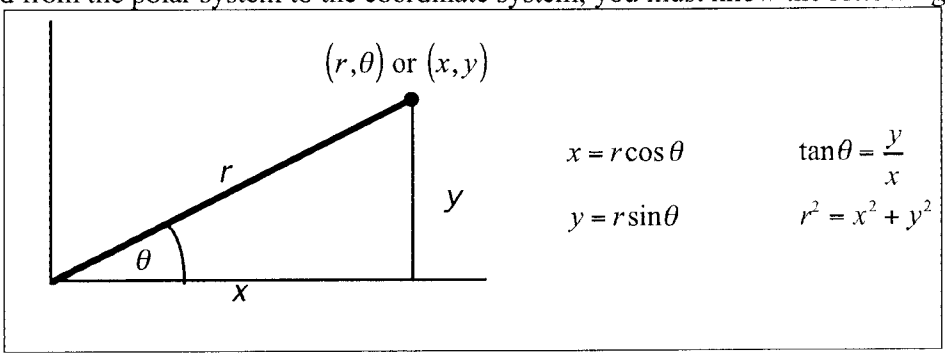
c. $(2,-90°)$
$\boxed{(2,270°),(-2,90°)}$

d. $\left(1,\frac{5\pi}{6}\right)$
$\boxed{\left(-1,\frac{11\pi}{6}\right),\left(1,-\frac{7\pi}{6}\right)}$

e. $\left(-8,\frac{\pi}{6}\right)$
$\boxed{\left(8,\frac{7\pi}{6}\right),\left(8,\frac{-5\pi}{6}\right)}$

f. $\left(-\frac{3}{2},-\frac{5\pi}{3}\right)$
$\boxed{\left(\frac{3}{2},\frac{4\pi}{3}\right),\left(\frac{3}{2},\frac{-2\pi}{3}\right)}$

To convert to and from the polar system to the coordinate system, you must know the following relationships.



$x = r\cos\theta$ $\qquad \tan\theta = \frac{y}{x}$

$y = r\sin\theta$ $\qquad r^2 = x^2 + y^2$

Example 2) Convert the following polar points to rectangular coordinates.

a. $(6,90°)$
$\boxed{(0,6)}$

b. $(4,60°)$
$\boxed{(2,2\sqrt{3})}$

c. $(10,225°)$
$\boxed{(-5\sqrt{2},-5\sqrt{2})}$

d. $(5, \pi)$
$\boxed{(-5,0)}$

e. $\left(2\sqrt{3},\frac{\pi}{6}\right)$
$\boxed{(3,\sqrt{3})}$

f. $\left(\frac{5}{2},\frac{5\pi}{3}\right)$
$\boxed{\left(\frac{5}{2},\frac{-5\sqrt{3}}{2}\right)}$

Example 3) Convert the following rectangular points to polar coordinates.

a. $(-5, -5)$
$\boxed{(5,225°)}$

b. $(0, -2)$
$\boxed{(2,270°)}$

c. $\left(1,-\sqrt{3}\right)$
$\boxed{(2,300°)}$

d. $(-7,0)$
$(7,180°)$

e. $(5,12)$
$\boxed{(13,67.38°)}$

f. $(6,-3)$
$\boxed{(3\sqrt{5},43°)}$

Example 4 -- Convert the following rectangular equations to polar equations.

a) $x^2 + y^2 = 25$

b) $(x+2)^2 + y^2 = 4$

c) $y = 3$

d) $x = 3$

e) $xy = 1$

f) $2x - 3y - 2 = 0$

Example 5 -- Convert the following polar equations to rectangular equations.

a) $r = 2$

$\sqrt{x^2+y^2} = 2$

$x^2+y^2 = 4$

b) $\theta = \frac{2\pi}{3}$

$\tan\theta = \tan\frac{2\pi}{3}$

$\frac{y}{x} = -\sqrt{3}$

$y = -x\cdot\sqrt{3}$

c) $r = 4\sec\theta$

$r = \frac{4}{\cos\theta}$

$r\cos\theta = 4$

$x = 4$

d) $r = -2\csc\theta$

$r = \frac{-2}{\sin\theta}$

$r\sin\theta = -2$

$y = -2$

e) $r = \frac{12}{3\sin\theta - 4\cos\theta}$

$1 = \frac{12}{3r\sin\theta - 4r\cos\theta}$

$1 = \frac{12}{3y - 4x}$

$3y - 4x = 12$

$3y = 4x + 12$

$y = \frac{4}{3}x + 4$

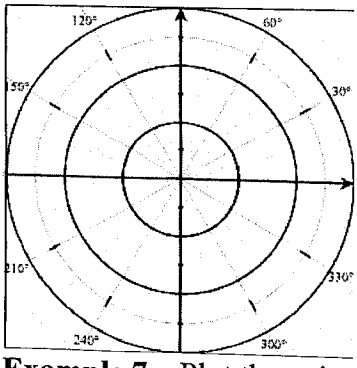
f) $r = \frac{3}{1+\sin\theta}$

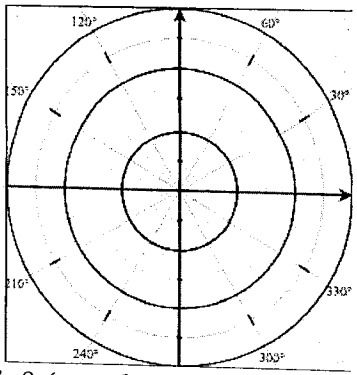
$1 = \frac{3}{r + r\sin\theta}$

$\sqrt{x^2+y^2} + y = 3$

Example 6 – Plot the points and sketch the graph of the polar equation: $r = 3\cos\theta$ (round to 1 decimal place)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°





Example 7 – Plot the points and sketch the graph of the polar equation: $r = 3 + 2\sin\theta$ (round to 1 decimal place)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°