Precalculus Unit 7 Test Review—Vectors, Polars, & Parametrics

In the following exercises, eliminate the parameter and describe the graph of the function.

1. $x = t$ & $y = 2t - 1$
   $y = 2x - 1$ line $m=2$ $b=-1$

2. $x = t + 3$ & $y = t^2$
   $y = (x-3)^2$ parabola opens up vertex $(3,0)$

3. $x = t - 2$ & $y = \dfrac{t}{t-2}$
   $y = \dfrac{x+2}{x}$ rational function

4. $x = t^5$ & $y = 5\ln t$
   $y = \ln x$ natural log function

5. A dart is thrown upward from 6 ft. high with an initial velocity of 18 feet/sec at an angle of elevation of 41°

a. Write a parameterization describing the position of the dart at time t.

$x(t) =$ $18\cos 41^\circ t$ $y(t) =$ $-16t^2 + 18\sin 41^\circ t + 6$

b. Approximately how long will it take for the dart to hit the ground?

$t \approx 1.08$ sec

c. Find the approximate maximum height of the dart.

8.18 ft

d. How long will it take for the dart to reach maximum height?

0.37 sec

6. A golfer hits a ball with an initial velocity of 90 ~~mph~~ ft/sec at angle of elevation of 64°.

a, Write a parametric equation that describes the position of the ball at time t.

$x(t) =$ $90\cos 64^\circ t$ $y(t) =$ $-16t^2 + 90\sin 64^\circ t$

b. Approximately how long will it take for the ball to hit the ground? 5.06 sec

c. Find the approximate maximum height of the ball. 102.2 ft

d, The green is 150 yards away. Will the ball reach the green? Explain. no – ball hits ground before @ $t = 5.06$ sec

$\Rightarrow 450$ ft $90\cos 64^\circ t = 450$

$t = 11.4$ sec

7. Convert the following polar points to rectangular coordinates.

a. $\left(6, \dfrac{\pi}{2}\right)$
$(0, 6)$

b. $(3, 120°)$
$\left(-\dfrac{3}{2}, \dfrac{3\sqrt{3}}{2}\right)$

c. $(10, 72°)$ **Need calc for this one
$(3.09, 9.51)$

8. Convert the following rectangular points to polar coordinates *answers may vary*

a. $(-3, 3)$
$\left(-3\sqrt{2}, -\dfrac{\pi}{4}\right)$
or
$\left(3\sqrt{2}, \dfrac{3\pi}{4}\right)$

b. $\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$
$\left(1, -\dfrac{\pi}{3}\right)$
or
$\left(-1, \dfrac{2\pi}{3}\right)$

c. $(7, -24)$
$(25, -73.74^\circ)$

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9. For each of the following rectangular equations, change it to polar form.

a. $5x - y = 7$ **b.** $(x-1)^2 + y^2 = 1$ **c.** $x^2 + y^2 + 4x = 0$

$r = \dfrac{7}{5\cos\theta - \sin\theta}$ $r = 2\cos\theta$ $r = -4\cos\theta$

10. For each of the following polar equations, change it to rectangular form.

a. $r = 4$ **b.** $r = 8\csc\theta$ **c.** $r = \dfrac{5}{2\sin\theta - \cos\theta}$

$x^2 + y^2 = 4$ circle $C(0,0)$ $r=4$ $y = 8$ horiz. line $2y - x = 5 \Rightarrow y = \frac{1}{2}x + \frac{5}{2}$ line $m=\frac{1}{2}$ $b=\frac{5}{2}$

11. Find component form and the magnitude of the vector **v** with initial point P and terminal point Q.

$P(-3,7)$ $Q(2,-1)$ $\vec{v} = \overrightarrow{PQ} = \langle 5,-8\rangle$

$|\vec{v}| = \sqrt{89}$

12. Given the vectors $\mathbf{u} = \langle 1,-3\rangle$, $\mathbf{v} = \langle 3,9\rangle$, find the following:

a. $\mathbf{u}+\mathbf{v}$ **b.** $\mathbf{u}-\mathbf{v}$ **c.** $8\mathbf{u} - 5\mathbf{v}$ **d.** $\mathbf{u}\cdot\mathbf{v}$

$\langle 4,6\rangle$ $\langle -2,-12\rangle$ $\langle -7,-69\rangle$ -24

e. $\text{proj}_{\mathbf{v}}\mathbf{u}$ **f.** Write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$) **g.** The angle between **u** and **v**

$\left\langle -\frac{4}{5}, \frac{-12}{5}\right\rangle$ $\vec{u} = \left\langle -\frac{4}{5}, \frac{-12}{5}\right\rangle + \left\langle \frac{9}{5}, \frac{-3}{5}\right\rangle$ $143.13°$

13. Find a unit vector in the direction of the vector $\mathbf{v} = \langle 2,-13\rangle$, and show that it has length 1.

$\left\langle \frac{2}{\sqrt{173}}, \frac{-13}{\sqrt{173}}\right\rangle$ magnitude $= \sqrt{\left(\frac{2}{\sqrt{173}}\right)^2 + \left(\frac{-13}{\sqrt{173}}\right)^2} = \sqrt{\frac{4}{173} + \frac{169}{173}}$

$= \sqrt{1} = 1$

14. Let **u** be the vector with initial point (13, 5) and terminal point (–12, 5) and let $\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$. Write the following as a linear combination of **i** and **j**. $\vec{u} = \langle -25,0\rangle$ $\vec{v} = \langle 8,6\rangle$

a. $-2\mathbf{u}$ **b.** $\mathbf{u} - 2\mathbf{v}$ $2\vec{v} = \langle 16,12\rangle$

$\langle 50,0\rangle$

$50i + 0j$

$\langle -41,-12\rangle$

$-41i - 12j$

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15. Write the vector **v** given its magnitude and direction angle: $|\mathbf{v}| = 13 \quad \theta = 60°$

$\langle 13\cos 60°, 13\sin 60° \rangle = \langle \frac{13}{2}, \frac{13\sqrt{3}}{2} \rangle$

16. A plane is flying on a bearing of 320° at 375 mph. A wind is blowing with the bearing 305° at 45mph.

a. Write a vector (in component form) of the velocity produced by the airplane alone.

p = $\langle 375\cos 130°, 375\sin 130° \rangle$

b. Write a vector (in component form) of the velocity of the wind.

w = $\langle 45\cos 145°, 45\sin 145° \rangle$

c. Write a vector (in component form) of the actual velocity of the plane.

v = $\langle -277.91, 313.08 \rangle$

d. Find the actual speed and direction angle (not the bearing) of the plane.

speed = 418.63 mph θ = 131.59°

17. Find the vector projection **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle 3, -9 \rangle$ & $\mathbf{v} = \langle -1, 7 \rangle$

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle \frac{33}{25}, \frac{-231}{25} \rangle$

$\mathbf{u} = \langle \frac{33}{25}, \frac{-231}{25} \rangle + \langle \frac{42}{25}, \frac{6}{25} \rangle$

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8. Match each polar equation with its graph below.

K 1) $r = 2.5 + 2.5\sin\theta$

H 2) $r = 3$

M 3) $r = 3.5\sin(3\theta)$

P 4) $r = 4.5\sin(2\theta)$

L 5) $r = 4.5\cos(2\theta)$

A 6) $r = 1.5 + 2\cos\theta$

B 7) $r = -3\sin\theta$

N 8) $r = 2 - \sin\theta$

G 9) $r^2 = 16\sin(2\theta)$

O 10) $r = 4\cos(5\theta)$

E 11) $r = 3.5\cos(3\theta)$

F 12) $r = 2.5 - 2.5\cos\theta$

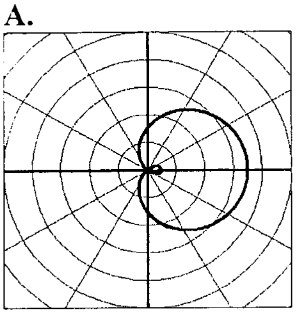
I 13) $r = 3\cos\theta$

J 14) $r = 1 + 4\sin\theta$

C 15) $r = 4.5\sin(6\theta)$

D 16) $r = \frac{1}{2}\theta$

A.



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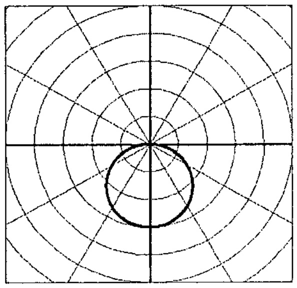

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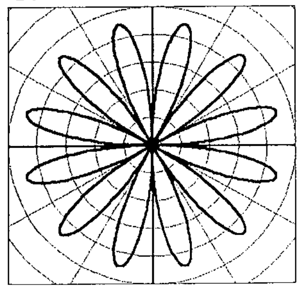

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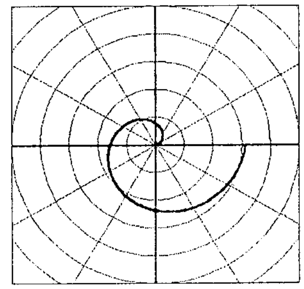

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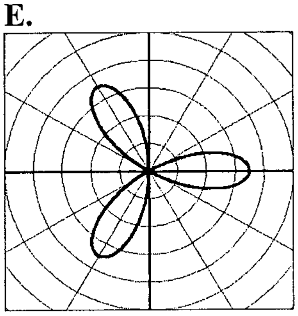

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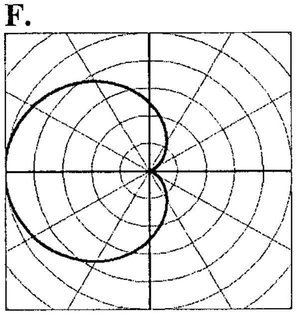

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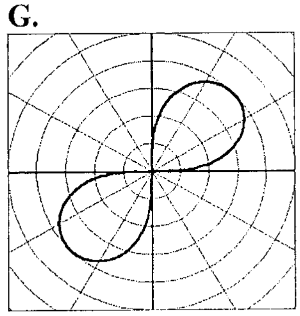

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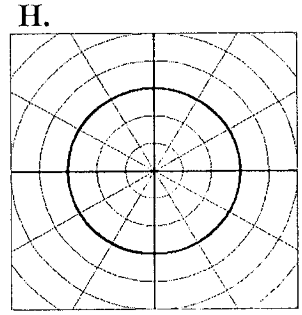

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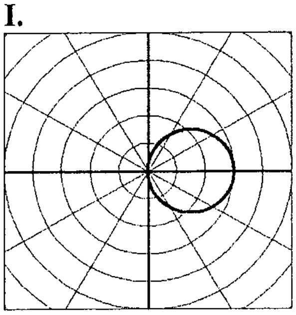

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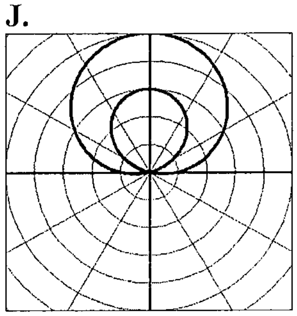

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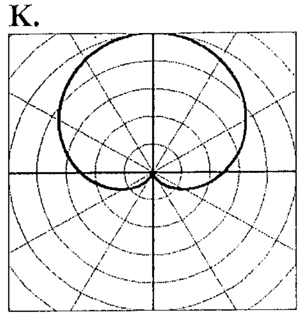

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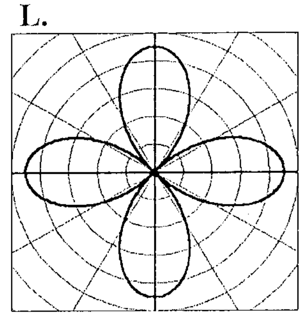

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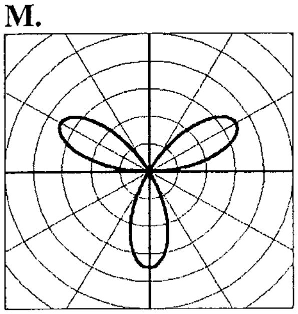

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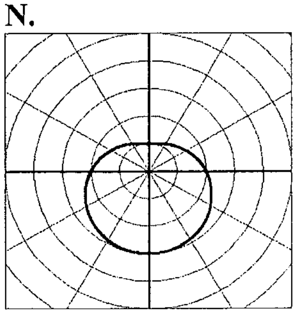

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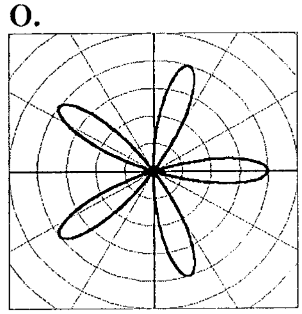

P.