

Find the sum of the infinite geometric series.

37. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

38. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

39. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

40. $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots$

41. $\frac{1}{3^6} + \frac{1}{3^8} + \frac{1}{3^{10}} + \frac{1}{3^{12}} + \dots$

42. $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \dots$

43. $-\frac{100}{9} + \frac{10}{3} - 1 + \frac{3}{10} + \dots$

44. $\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \frac{1}{4} + \dots$

Express the repeating decimal as a fraction.

45. $0.7777\dots$

46. $0.25\overline{3}$

47. $0.030303\dots$

48. $2.11\overline{25}$

49. $0.11\overline{2}$

50. $0.123123123\dots$

51. A certain ball rebounds to half the height from which it is dropped. Use an infinite geometric series to approximate the total distance the ball travels, after being dropped from 1 meter above the ground, until it comes to rest.

REVIEW (9.4)--- Mixed Sequences & Series

① Determine the 65th term for the sequence 63, 59, 55, 51... $a_1 = 63$ $d = -4$ $a_n = 63 - 4(n-1)$
 $a_{65} = 63 - 4(65-1) = -1$

② Determine the common difference for the sequence $\frac{2}{5}, \frac{16}{15}, \frac{26}{15}, \frac{12}{5}$ $d = \frac{2}{3}$

③ The ninth term of an arithmetic sequence is 10.6 and the first term is 5. Find the 25th term. $10.6 = 5 + d(9)$
 $d = .7$
 $a_n = 5 + .7(n)$
 $a_{25} = 5 + .7(25) = 21.8$

4. The 7th term of an arithmetic sequence is 2 and the 16th term is -25. Find the 95th term.

⑤ Find the sum of the arithmetic series $-9 - 5 - 1 + 3 + \dots + 283$ $283 = -9 + 4(n-1)$
 $S_{74} = \frac{74}{2}(-9 + 283) = 10138$ $n = 74$

⑥ A stack of books is on display in a bookstore. There are 50 books on the first layer, 47 in the second layer, 44 in the third, and so on. There are 17 layers of books. $50, 47, 44, \dots, 2$

a. How many books are in the stack? $S_{17} = \frac{17}{2}(50+2) = 442$ $d = -3$ $a_1 = 50$
 b. How many books are in layers 8 through layer 14 (inclusively)? $a_8 = 50 - 3(8-1) = 29$ $a_{17} = 50 - 3(17-1) = 2$
 c. How many books are in row 7? $a_7 = 50 - 3(7-1) = 32$ $a_{14} = 50 - 3(14-1) = 11$

⑦ Find the 9th term in the sequence 1200, 300, 75, 18.75... $r = \frac{1}{4}$ $a_1 = 1200$ $a_9 = 1200(\frac{1}{4})^{9-1} = \frac{75}{4096}$ $S_7 = \frac{7}{2}(29+11) = 140$

⑧ $\sum_{n=1}^7 243(-\frac{1}{3})^{n-1}$ $243 + -81 + 27 + \dots$ $r = -\frac{1}{3}$ $a_1 = 243$
 $S_7 = 243 \cdot \frac{1 - (-\frac{1}{3})^7}{1 - (-\frac{1}{3})} = \frac{547}{3}$

⑨ $\sum_{n=1}^{37} (4n+3)$ $7 + 11 + 15 + \dots + 151$ $d = 4$ $a_1 = 7$ $a_{37} = 151$
 $S_{37} = \frac{37}{2}(7+151) = 2923$

⑩ $\sum_{n=8}^{48} -4n = -32 + -36 + -40 + \dots$ $d = -4$ 41 terms
 $S_{41} = \frac{41}{2}(-32 + -41) = -4592$

1. Which term in the geometric sequence 4, 12, 36, ..., is 708588?

$r = 3 \quad a_1 = 4$

$708588 = 4(3)^{n-1}$
 $177147 = 3^{n-1}$
 $177147 = \frac{3^n}{3}$

12. Write a recursive formula for the sequence 8, 10, 12, 14, 16, ...

$a_1 = 8 \quad a_n = a_{n-1} + 2$

13. Use sigma notation to express the following series.

a. 0.5-1+2-4+8-16+32-64+128-256 geom. $r = -2 \quad a_1 = 0.5$

b. 7+7+7+7+7+7+7+7

c. 2+5+8+11+14+17 arith.

d. 20+10+5+... $d = 3 \quad a_1 = 2$
 $\sum_{n=1}^{\infty} [2+3(n-1)]$

$\sum_{n=1}^{\infty} 0.5(-2)^{n-1}$

$3^n = 531441$
 $n \cdot \log 3 = \log 531441$
 $n = 12$

14. Find the 75th term defined by the explicit formula $t_n = -n + 4$.

15. A ball is dropped from a height of 120 ft. The elasticity of the ball is such that it rebounds one-half the distance it has fallen.

- a. How high does the ball rebound on the nth bounce?
- b. How high does the ball rebound on the sixth bounce?
- c. How far will the ball travel before it stops?

16. Find the seventh term of the sequence defined by the recursive formula $t_n = 3t_{n-1} - 4n$ with $t_1 = 5$.

17. The end of a swinging pendulum 40 cm long travels 30 cm on its first swing. Each swing is 80 percent as long as the preceding one. How far will the pendulum travel before coming to rest?

18. A ball is dropped from 200 feet. One each bounce, the ball rises to a height three-fourths of the previous bounce. How far will the ball travel before it stops bouncing?

$200 + 150 + 112.5 + 84.375 + \dots$
 $r = \frac{3}{4} \quad |\frac{3}{4}| < 1$ converges $S = \frac{200}{1 - \frac{3}{4}} = 800$

19. Find the sum of the infinite geometric series $9+6+4+\dots$

$r = \frac{2}{3} \quad |\frac{2}{3}| < 1$ converges $S = \frac{9}{1 - \frac{2}{3}} = 27$

20. Express the following decimals as fractions.

- a. 0.31
- b. 0.8
- c. 0.432
- d. 0.2410

21. Find the first term in a geometric series where the sum of the first 7 terms is 21589 and the common ratio is 4.

$S_n = a_1 \cdot \frac{1-r^n}{1-r} \quad 21589 = a_1 \cdot \left(\frac{1-4^7}{1-4}\right) \quad a_1 = \frac{21589}{5461}$

22. Find the seventh term of the sequence defined by the recursive formula $t_n = 3t_{n-1} + 2$ with $t_1 = 4$.

4, 14, 44, 134, 404, 1214, 3644

$3 \cdot 4 + 2 = 14$
 $3 \cdot 14 + 2 = 44$
 $3 \cdot 44 + 2 = 134$
 $3 \cdot 134 + 2 = 404$
 $3 \cdot 404 + 2 = 1214$
 $3 \cdot 1214 + 2 = 3644$

23. Is 33867 a term of the sequence 3, 9, 15, 21, ...? If so, which term?

24. Is 262144 a term of the sequence 4, -16, 64, -256, ...? If so, which term?

25. A large amphitheater has 65 rows of seats with 20 seats in the first row, 24 seats are in the second row, 28 in the third row and so on.

- a. How many seats are in row 60?
- b. How many seats are in the theater?
- c. How many seats are there in rows 20 through 59 (inclusively)?

$\frac{5x+7}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$
 $5x+7 = A(x-1) + B(x+3)$
 $x=1: 12 = 4B \quad B=3$
 $x=-3: -8 = -4A \quad A=2$
 $\frac{2}{x+3} + \frac{3}{x-1}$