AP Calculus

Practice   Volume: The Shell Method

1) Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

a) $y = x$, $y = 0$, $x = 2$ b) $y = x^2$, $y = 0$, $x = 2$ c) $y = 4x - x^2$, $x = 0$, $y = 4$

$$2\pi \int_0^2 x \cdot x \, dx = \frac{16\pi}{3}$$ $$2\pi \int_0^2 x \cdot x^2 \, dx = 8\pi$$ $$2\pi \int_0^2 x(4 - (4x - x^2)) \, dx = \frac{8\pi}{3}$$

2) Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

a) $y = x$, $y = 0$, $x = 2$ b) $y = x^3$, $x = 0$, $y = 8$

$$2\pi \int_0^2 y (2 - y) \, dy = \frac{8\pi}{3}$$ $$2\pi \int_0^8 y \cdot \sqrt[3]{y} \, dy = \frac{768\pi}{7}$$

3) Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the indicate line.

a) $y = x^2$, $y = 4x - x^2$ about the line $x = 4$ $$2\pi \int_0^2 (4 - x)(4x - x^2 - x^2) \, dx = 16\pi$$

b) $y = 4x - x^2$, $y = 0$ about the line $x = 5$ $$2\pi \int_0^4 (5 - x)(4x - x^2) \, dx = 64\pi$$

4) Use any method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = x^3$, $y = 0$, and $x = 2$ about the indicated line.

a) the x-axis   b) the y-axis   c) the line $x = 4$

$$2\pi \int_0^8 y (2 - \sqrt[3]{y}) \, dy$$ $$2\pi \int_0^2 x \cdot x^3 \, dx$$ $$2\pi \int_0^2 (4 - x) x^3 \, dx$$

$$\frac{128\pi}{7}$$ $$\frac{64\pi}{5}$$ $$\frac{96\pi}{5}$$

10