

## Slope Fields - Non Calculator

### PART I

- 1) Match the slope fields below with one of the following 8 first-order equations:

$$(1) \frac{dy}{dx} = x - 1$$

$$(2) \frac{dy}{dx} = x + 1$$

$$(3) \frac{dy}{dx} = y + 1$$

$$(4) \frac{dy}{dx} = 1 - y$$

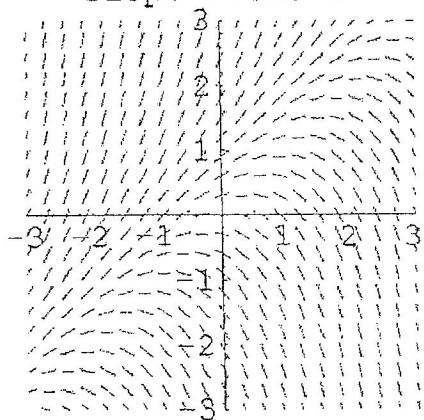
$$(5) \frac{dy}{dx} = y^2 + y$$

$$(6) \frac{dy}{dx} = y(y^2 - 1)$$

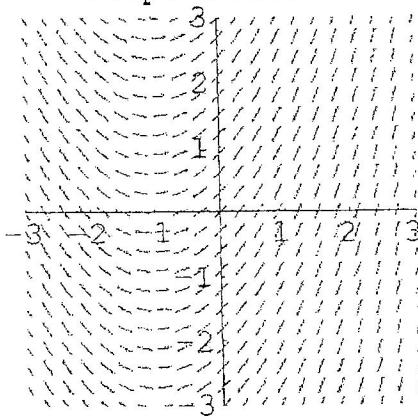
$$(7) \frac{dy}{dx} = y - x$$

$$(8) \frac{dy}{dx} = y + x$$

Slope Field A



Slope Field B



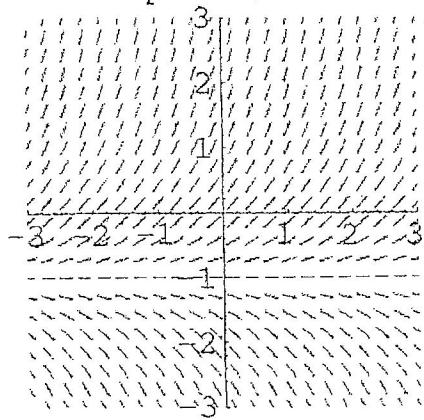
1) SF A = 7

SF B = 2

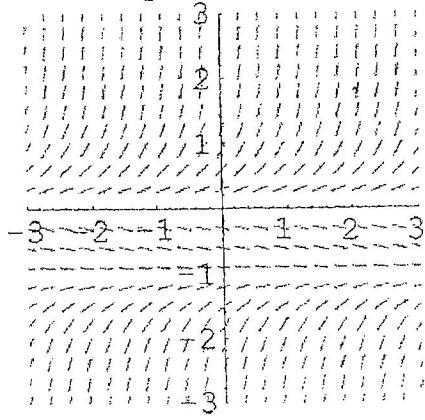
SF C = 3

SF D = 5

Slope Field C

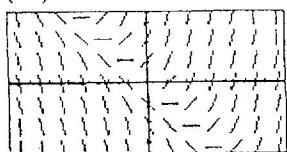


Slope Field D

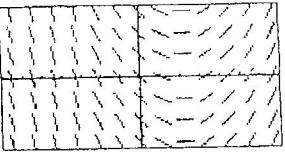


- 2) Fill in the appropriate letter slope field in the blanks below next to its matching differential equation.

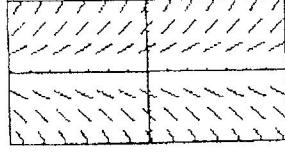
(A)



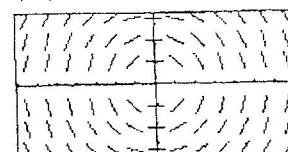
(B)



(C)



(D)



B  $\frac{dy}{dx} = .5x - 1$

C  $\frac{dy}{dx} = .5y$

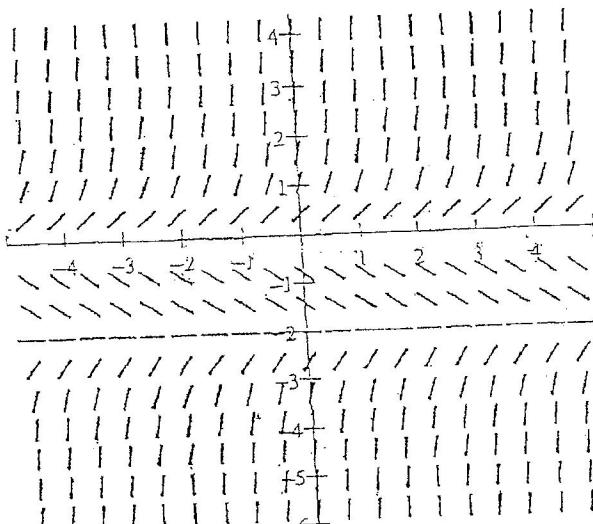
D  $\frac{dy}{dx} = -\frac{x}{y}$

A  $\frac{dy}{dx} = x + y$

- 3) Which statement is true about the solutions  $y(x)$ , of a differential equation whose slope field is shown below?

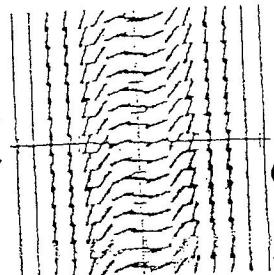
- I. If  $y(0) > 0$  then  $\lim_{x \rightarrow \infty} y(x) \approx 0$ .
- II. If  $-2 < y(0) < 0$  then  $\lim_{x \rightarrow \infty} y(x) \approx -2$ .
- III. If  $y(0) < -2$  then  $\lim_{x \rightarrow \infty} y(x) \approx -2$ .

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III



- 4) The slope field for the differential equation  $dy/dx = f(x)$  is shown below for  $-4 < x < 4$  and  $-4 < y < 4$ . Which statement is true for all possible solutions of the differential equation?

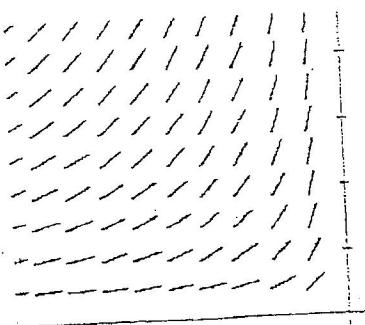
- I. For  $x < 0$  all solutions are <sup>increasing</sup> decreasing.
- II. All solutions level off near the y-axis.
- III. For  $x > 0$  all solutions are increasing.



- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

- 5) The figure shows part of a slope field for a differential equation in the second quadrant. Based on the figure, which statement is true?

- I. As  $x$  approaches zero from the left,  $y$  increases without bound.
- II. As  $x$  decreases without bound,  $y$  decreases without bound.
- III. For points in the second quadrant,  $dy/dx > 0$ .



- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

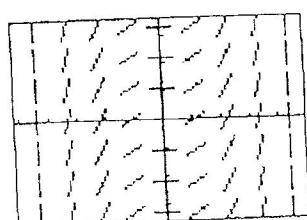
- 6) The slope field for a certain differential equation is shown below. Which of the following could be a specific solution to that differential equation?

(A)  $y = \sin x$       (B)  $y = \cos x$

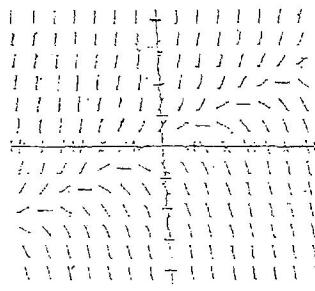
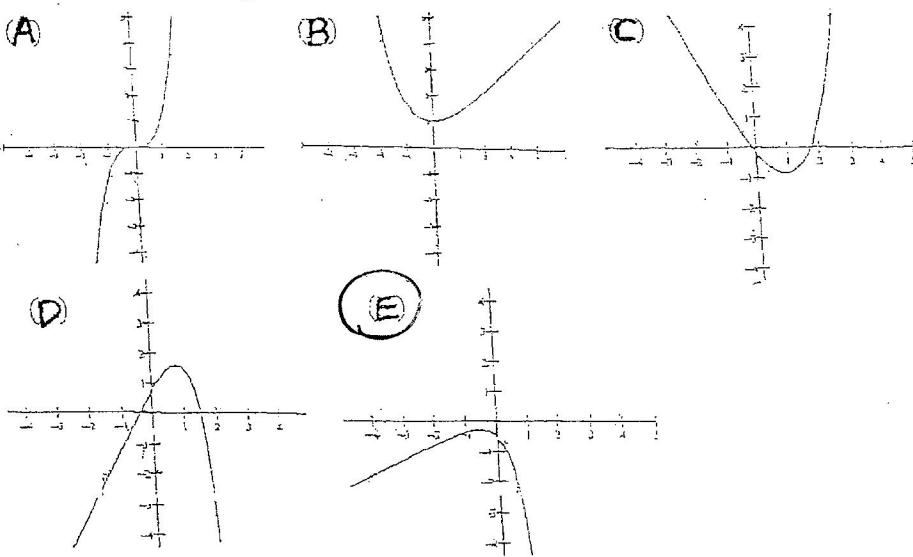
(C)  $y = x^2$

(D)  $y = \frac{1}{6}x^3$

(E)  $y = \ln x$



- 7) Which of the graphs below could be the graph of the solution of the differential equation whose slope field is shown here?

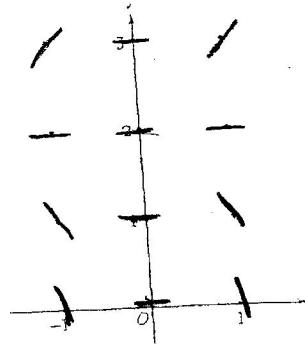


PART II: Show all work and explain where necessary to receive full credit

8) Consider the differential equation  $\frac{dy}{dx} = x^4(y-2)$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

x	y	$\frac{dy}{dx}$
1	3	1
-1	3	1
1	1	-1
-1	1	-1
1	0	-2
-1	0	-2



- (b) While the slope field in part (a) is drawn only at twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

$$\frac{dy}{dx} = x^4(y-2) < 0$$

when  $y-2 < 0$

$y < 2$

- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 0$ .

$$\int \frac{dy}{y-2} = \int x^4 dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$\text{Given } (0, 0) \therefore \ln|0-2| = \frac{1}{5}(0)^5 + C$$

$$\ln 2 = C$$

$$\ln|y-2| = \frac{1}{5}x^5 + \ln 2$$

$$e^{\ln|y-2|} = e^{\frac{1}{5}x^5 + \ln 2}$$

$$y-2 = e^{\frac{1}{5}x^5} \cdot e^{\ln 2}$$

$$y = 2e^{\frac{1}{5}x^5} + 2$$

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**Homework – Euler's Method**

1. Given the differential equation  $\frac{dy}{dx} = x + 2$  with the initial point  $(0, 1)$ , use Euler's method with a step size of 0.1 to approximate the value of  $f(0.2)$ .

x	y	$\frac{dy}{dx}$
0	1	2
0.1	1.2	2.1
0.2	1.41	

$$1 + 2(0.1) = 1.2$$

$$1.2 + 2.1(0.1) = 1.41$$

$$f(0.2) \approx 1.41$$

**Multiple-Choice Questions***No calculator is allowed for these questions.*

1.  $\frac{dy}{dx} = \frac{x}{1+x^2}$  and  $y(0) = 0$ . Using Euler's method with step  $\Delta x = 0.2$ ,  $y(0.2)$  is approximately

- (A) 0  
 (B) 0.0196  
 (C) 0.192  
 (D) 0.2  
 (E) 0.888

x	y	$\frac{dy}{dx}$
0	0	0
0.2	0	

$$0 + 0(0.2) = 0$$

$$y(0.2) \approx 0$$

2.  $\frac{dy}{dx} = x + y$  and  $y(0) = 1$ . Using Euler's method with step  $\Delta x = 0.5$ , approximate the value of  $y(0.5)$ .

- (A) 0  
 (B) 1  
 (C) 1.5  
 (D) 1.797  
 (E) 2

x	y	$\frac{dy}{dx}$
0	1	1
0.5	1.5	

$$1 + 1(0.5) = 1.5$$

$$y(0.5) \approx 1.5$$

2. Given the differential equation  $\frac{dy}{dx} = 3x$  with the initial condition  $f(1) = 1$ , use Euler's method with a step size of 0.1 to approximate the value of  $f(1.2)$ .

x	y	$\frac{dy}{dx}$
1	1	3
1.1	1.3	3.3
1.2	1.63	

$$1 + 3(0.1) = 1.3$$

$$1.3 + 3.3(0.1) = 1.63$$

$$f(1.2) \approx 1.63$$

3.  $x^2 \frac{dy}{dx} = y - 1$  and  $y(1) = 2$ . Use Euler's method with step  $\Delta x = 0.1$  to approximate  $y(1.2)$ .

- (A) 2  
 (B) 2.091  
 (C) 2.1  
 (D) 2.181  
 (E) 2.191

x	y	$\frac{dy}{dx}$
1	2	1
1.1	2.1	1.1
1.2	2.191	

$$2 + 1(0.1) = 2.1$$

$$2.1 + 1.1(0.1) = 2.11$$

$$2.11 + 1.1(0.1) = 2.191$$

$$y(1.2) \approx 2.191$$

4.  $(x^2 + 1) \frac{dy}{dx} = y - 1$  and  $y(0) = 3$ . What is the approximate value of  $y(0.1)$  using Euler's method with step  $\Delta x = 0.1$ ?

- (A) -0.891  
 (B) 3  
 (C) 3.2  
 (D) 3.21  
 (E) 3.418

x	y	$\frac{dy}{dx}$
0	3	2
0.1	3.2	

$$3 + 2(0.1) = 3.2$$

$$y(0.1) \approx 3.2$$