ICM Name KEY Fall 2017

Review Unit 1: Election Theory & Fair Division

Aubrey, Ben, Charlie, Derek, and Emily are running for class president. Use the following preference schedules to determine the winner as indicated.

# of votes	20	22	12	9
First Choice	A	B	E	D
Second	C	D	C	E
Third Choice	D	C	D	C
Fourth	E	A	A	A
Fifth Choice	B	E	B	B

1. Determine the winner using a Borda count. D
2. Determine the plurality winner. B
3. Determine the runoff winner. A
4. Determine the sequential runoff winner. E
5. Determine the Condorcet winner, if there is one. C
6. Suppose that this election is conducted by the approval method and all the voters decide to approve of the first **two** choices on the preference schedule. Determine the approval winner. C

7. Anne, Beth, and Jay are heirs to an estate that includes a computer, a used car, and a stereo. Each heir has submitted bids for the items in the estate as summarized in the following table. Use the method of sealed bids to determine the item(s) that each person receives and the final amount of cash that he or she receives or pays.

	Anne	Beth	Jay
Computer	$1,800	$1,500	$1,650
Car	$2,600	$2,400	$2,000
Stereo	$1,000	$800	$1,200

< see work >

8. Your class is voting on a favorite soda. After everyone ranks the sodas, someone suggests that the names of the sodas be placed in a hat and the group ranking be determined by drawing them from the hat. Which of Arrow's conditions does this method violate?

nondictatorship & indiv. SOV.

9. Consider a situation in which voters A, B, and C have 1, 3, and 2 votes respectively and 4 votes are needed to pass an issue.
   a. List all coalitions and their vote totals. Determine if each coalition is a winning one.
   b. Find the Banzhaf power index of each voter.
   c. Find the Shapley-Shubik power distribution of each voter.

see work

10. States A, B, C, and D have populations of 647, 247, 106, and 360 respectively. There are 85 seats to be apportioned among them.

   a. What is the Standard Divisor? $\frac{1360}{85} = 16$

   b. Determine the apportionment using the methods of Hamilton, Jefferson, Webster, Huntington-Hill. (Fill in the chart below).

		SD = 16			MD = 15.5	
state	population	SQ	LQ	Hamilton Apportionment	MQ	Jefferson Apportionment
A	647	40.438	40	40	41.742	41
B	247	15.438	15	15	15.94	15
C	106	6.625	6	7	6.84	6
D	360	22.5	22	23	23.23	23
TOTAL	1360	85	83	85		85

		MD =			MD =	
state	UQ	MQ	Webster Apportionment	GM	MQ	Huntington-Hill Apportionment
A	41		40	40.497		40
B	16		15	15.492		15
C	7		7	6.481		7
D	23		23	22.494		23
TOTAL	~~		85			85

1. A: $20(5)+0(4)+0(3)+43(2)+0(1)=186$

B: $22(5)+0(4)+0(3)+0(2)+41(1)=151$

C: $0(5)+32(4)+31(3)+0(2)+0(1)=221$

D: $9(5)+22(4)+32(3)+0(2)+0(1)=229$

E: $12(5)+9(4)+0(3)+20(2)+22(1)=158$

D wins

2.

A	20
B	22
C	0
D	9
E	12

B wins

3.

A	20
B	22
C	0
D	9
E	12

A	41
B	22

A wins

4.

A	20
B	22
C	0
D	9
E	12

A	20
B	22
D	9
E	12

A	20
B	22
E	21

B	22
E	41

E wins

5.

(A)	B
41	22

A	(C)
20	43

A	(D)
20	43

(A)	E
42	21

B	(C)
22	41

B	(D)
22	41

B	(E)
22	41

(C)	D
32	31

(C)	E
42	21

(D)	E
51	12

C wins

6. A 20 = 20
B 22 = 22
C 20+12 = 32 C wins
D 22+9 = 31
E 12+9 = 21

7.

	A	B	J
total bids	5400	4700	4850
fair share	1800	1566.67	1616.67

A: 4400 − 1800 = 2600
B: 0 − 1566.67 = −1566.67
J: 1200 − 1616.67 = −416.67

2600
−1983.34
616.66 surplus
÷ 3
205.55 to each person

Anne: computer, car, pays $2394.45
Beth: gets $1772.22
Jay: stereo, gets $622.22

8. nondictatorship & individual SoV. [4: 1,3,2]

9.

coalition	winning?	weight	critical voter(s)
A	no	1	—
B	no	3	—
C	no	2	—
AB	yes	4	A, B
BC	yes	5	B, C
AC	no	3	—
ABC	yes	6	B

Banzhaf
A: $\frac{1}{5}$
B: $\frac{3}{5}$
C: $\frac{1}{5}$

Shapley-Shubik

A **B** C	B **A** C	C A **B**
A C **B**	B **C** A	C **B** A

(pivotal player underlined)

A: $\frac{1}{6} = 16.\bar{6}\%$

B: $\frac{4}{6} = 66.\bar{6}\%$

C: $\frac{1}{6} = 16.\bar{6}\%$

10. see chart