5.

$$
\begin{array}{r}
x^2 - 4x + 12 \\
x^2 + 2x - 1 \overline{\big) x^4 - 2x^3 + 3x^2 - 4x + 6} \\
\underline{x^4 + 2x^3 - x^2 } \\
-4x^3 + 4x^2 - 4x \\
\underline{-4x^3 - 8x^2 + 4x } \\
12x^2 - 8x + 6 \\
\underline{12x^2 + 24x - 12} \\
-32x + 18
\end{array}
$$

$x) = (x^2 - 4x + 12)(x^2 + 2x - 1) - 32x + 18;$

$$\frac{f(x)}{x^2 + 2x - 1} = x^2 - 4x + 12 + \frac{-32x + 18}{x^2 + 2x - 1}$$

6.

$$
\begin{array}{r}
x^2 - 3x + 5 \\
x^2 + 1 \overline{\big) x^4 - 3x^3 + 6x^2 - 3x + 5} \\
\underline{x^4 + x^2 } \\
-3x^3 + 5x^2 - 3x \\
\underline{-3x^3 - 3x } \\
5x^2 + 5 \\
\underline{5x^2 + 5} \\
0
\end{array}
$$

$f(x) = (x^2 - 3x + 5)(x^2 + 1); \dfrac{f(x)}{x^2 + 1} = x^2 - 3x + 5$

7. $\dfrac{x^3 - 5x^2 + 3x - 2}{x + 1} = x^2 - 6x + 9 + \dfrac{-11}{x + 1}$

-1	1	-5	3	-2
		-1	6	-9
	1	-6	9	-11

8. $\dfrac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$

$= 2x^3 + x^2 + 10x + 27 + \dfrac{82}{x - 3}$

3	2	-5	7	-3	1
		6	3	30	81
	2	1	10	27	82

9. $\dfrac{9x^3 + 7x^2 - 3x}{x - 10} = 9x^2 + 97x + 967 + \dfrac{9670}{x - 10}$

10	9	7	-3	0
		90	970	9670
	9	97	967	9670

10. $\dfrac{3x^4 + x^3 - 4x^2 + 9x - 3}{x + 5}$

$= 3x^3 - 14x^2 + 66x - 321 + \dfrac{1602}{x + 5}$

-5	3	1	-4	9	-3
		-15	70	-330	1605
	3	-14	66	-321	1602

11. $\dfrac{5x^4 - 3x + 1}{4 - x}$

$= -5x^3 - 20x^2 - 80x - 317 + \dfrac{-1269}{4 - x}$

4	-5	0	0	3	-1
		-20	-80	-320	-1268
	-5	-20	-80	-317	-1269

12. $\dfrac{x^8 - 1}{x + 2}$

$= x^7 - 2x^6 + 4x^5 - 8x^4 + 16x^3 - 32x^2 + 64x - 128$

$+ \dfrac{255}{x + 2}$

-2	1	0	0	0	0	0	0	0	-1
		-2	4	-8	16	-32	64	-128	256
	1	-2	4	-8	16	-32	64	-128	255

13. The remainder is $f(2) = 3$.

14. The remainder is $f(1) = -4$.

15. The remainder is $f(-3) = -43$.

16. The remainder is $f(-2) = 2$.

17. The remainder is $f(2) = 5$.

18. The remainder is $f(-1) = 23$.

19. Yes: 1 is a zero of the second polynomial.

20. Yes: 3 is a zero of the second polynomial.

21. No: when $x = 2$, the second polynomial evaluates to 10.

22. Yes: 2 is a zero of the second polynomial.

23. Yes: -2 is a zero of the second polynomial.

24. No: when $x = -1$, the second polynomial evaluates to 2.

25. From the graph it appears that $(x + 3)$ and $(x - 1)$ are factors.

-3	5	-7	-49	51
		-15	66	-51
1	5	-22	-17	0
		5	-17	
	5	-17	0	

$f(x) = (x + 3)(x - 1)(5x - 17)$

26. From the graph it appears that $(x + 2)$ and $(x - 3)$ are factors.

-2	5	-12	-23	42
		-10	44	-42
3	5	-22	21	0
		15	-21	
	5	-7	0	

$f(x) = (x + 2)(x - 3)(5x - 7)$

27. $2(x + 2)(x - 1)(x - 4) = 2x^3 - 6x^2 - 12x + 16$

28. $2(x + 1)(x - 3)(x + 5) = 2x^3 + 6x^2 - 26x - 30$

29. $2(x-2)\left(x-\frac{1}{2}\right)\left(x-\frac{3}{2}\right)$

$= \frac{1}{2}(x-2)(2x-1)(2x-3)$

$= 2x^3 - 8x^2 + \frac{19}{2}x - 3$

30. $2(x+3)(x+1)(x)\left(x-\frac{5}{2}\right)$

$= x(x+3)(x+1)(2x-5)$

$= 2x^4 + 3x^3 - 14x^2 - 15x$

31. Since $f(-4) = f(3) = f(5) = 0$, it must be that $(x+4)$, $(x-3)$, and $(x-5)$ are factors of f. So $f(x) = k(x+4)(x-3)(x-5)$ for some constant k.
Since $f(0) = 180$, we must have $k = 3$. So $f(x) = 3(x+4)(x-3)(x-5)$

32. Since $f(-2) = f(1) = f(5) = 0$, it must be that $(x+2)$, $(x-1)$, and $(x-5)$ are factors of f. So $f(x) = k(x+2)(x-1)(x-5)$ for some constant k.
Since $f(-1) = 24$, we must have $k = 2$, so $f(x) = 2(x+2)(x-1)(x-5)$

33. Possible rational zeros: $\frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$; 1 is a zero.

34. Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 3}$, or $\pm 1, \pm 2, \pm 7, \pm 14, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{7}{3}, \pm\frac{14}{3}$; $\frac{7}{3}$ is a zero.

35. Possible rational zeros: $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$, or $\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$; $\frac{3}{2}$ is a zero.

36. Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}$; $-\frac{4}{3}$ and $\frac{3}{2}$ are zeros.

37.

3	2	−4	1	−2
		6	6	21
	2	2	7	19

Since all numbers in the last line are ≥ 0, 3 is an upper bound for the zeros of f.

38.

5	2	−5	−5	−1
		10	25	100
	2	5	20	99

Since all values in the last line are ≥ 0, 5 is an upper bound for the zeros of $f(x)$.

39.

2	1	−1	1	1	−12
		2	2	6	14
	1	1	3	7	2

Since all values in the last line are ≥ 0, 2 is an upper bound for the zeros of $f(x)$.

40.

3	4	−6	−7	9	2
		12	18	33	126
	4	6	11	42	128

Since all values in the last line are ≥ 0, 3 is an upper bound for the zeros of $f(x)$.

41.

−1	3	−4	1	3
		−3	7	−8
	3	−7	8	−5

Since the values in the last line alternate signs, −1 is a lower bound for the zeros of $f(x)$.

42.

−3	1	2	2	5
		−3	3	−15
	1	−1	5	−10

Since the values in the last line alternate signs, −3 is a lower bound for the zeros of $f(x)$.

43.

0	1	−4	7	−2
		0	0	0
	1	−4	7	−2

Since the values in the last line alternate signs, 0 is a lower bound for the zeros of $f(x)$.

44.

−4	3	−1	−5	−3
		−12	52	−188
	3	−13	47	−191

Since the values in the last line alternate signs, −4 is a lower bound for the zeros of $f(x)$.

45. By the lower/upper bound tests, −5 is a lower bound and 5 is an upper bound. No zeros outside window.

−5	6	−11	−7	8	−34
		−30	205	−990	4910
	6	−41	198	−982	4876

5	6	−11	−7	8	−34
		30	95	440	2240
	6	19	88	448	2206

46. By the lower/upper bound tests, −5 is a lower bound and 5 is an upper bound. No zeros outside window.

−5	1	−1	0	21	19	−3
		−5	30	−150	645	−3320
	1	−6	30	−129	664	−3323

5	1	−1	0	21	19	−3
		5	20	100	605	3120
	1	4	20	121	624	−3117

47. Synthetic division shows that the lower/upper bound tests were not met. There *are* zeros not shown (appr −11.002 and 12.003), because −5 and 5 are not bour for zeros of $f(x)$.

−5	1	−4	−129	396	−8	3
		−5	45	420	−4080	20,440
	1	−9	−84	816	−4088	−20,443

5|	1	−4	−129	396	−8	3
		5	5	−620	−1120	−5640
	1	1	−124	−224	−1128	−5637

48. Synthetic division shows that the lower/upper bounds tests were not met. There *are* zeros not shown (approx. −8.036 and 9.038), because −5 and 5 are not bounds for zeros of $f(x)$.

−5|	2	−5	−141	216	−91	25
		−10	75	330	−2730	14,105
	2	−15	−66	546	−2821	14,130

5|	2	−5	−141	216	−91	25
		10	25	−580	−1820	−9555
	2	5	−116	−364	−1911	−9530

For #49–56, determine the rational zeros using a grapher (and the Rational Zeros Test as necessary). Use synthetic division to reduce the function to a quadratic polynomial, which can be solved with the quadratic formula (or otherwise). The first two are done in detail; for the rest, we show only the synthetic division step(s).

49. Possible rational zeros: $\dfrac{\pm1, \pm2, \pm3, \pm6}{\pm1, \pm2}$, or

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$. The only rational zero is $\frac{3}{2}$. Synthetic division (below) leaves $2x^2 - 4$, so the irrational zeros are $\pm\sqrt{2}$.

$\frac{3}{2}$|	2	−3	−4	6
		3	0	−6
	2	0	−4	0

50. Possible rational zeros: ±1, ±3, ±9. The only rational zero is −3. Synthetic division (below) leaves $x^2 - 3$, so the irrational zeros are $\pm\sqrt{3}$.

−3|	1	3	−3	−9
		−3	0	9
	1	0	−3	0

51. Rational: −3; irrational: $1 \pm \sqrt{3}$

−3|	1	1	−8	−6
		−3	6	6
	1	−2	−2	0

52. Rational: 4; irrational: $1 \pm \sqrt{2}$

4|	1	−6	7	4
		4	−8	−4
	1	−2	−1	0

53. Rational: −1 and 4; irrational: $\pm\sqrt{2}$

−1|	1	−3	−6	6	8
		−1	4	2	−8
	1	−4	−2	8	0

4|	1	−4	−2	8
		4	0	−8
	1	0	−2	0

54. Rational: −1 and 2; irrational: $\pm\sqrt{5}$

−1|	1	−1	−7	5	10
		−1	2	5	−10
	1	−2	−5	10	0

2|	1	−2	−5	10
		2	0	−10
	1	0	−5	0

55. Rational: $-\frac{1}{2}$ and 4; irrational: none

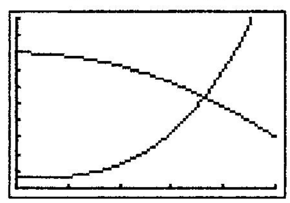
4|	2	−7	−2	−7	−4
		8	4	8	4
	2	1	2	1	0

$-\frac{1}{2}$|	2	1	2	1
		−1	0	−1
	2	0	2	0

56. Rational: $\frac{2}{3}$; irrational: about −0.6823

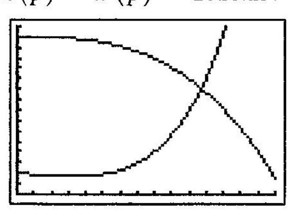
$\frac{2}{3}$|	3	−2	3	1	−2
		2	0	2	2
	3	0	3	3	0

57. The supply and demand graphs are shown on the window [0, 50] by [0, 100]. They intersect when p = \$36.27, at which point the supply and demand equal 53.7.



[0, 50] by [0, 100]

58. The supply and demand graphs, shown on the window [0, 150] by [0, 1600], intersect when p = \$106.99. There $S(p) = D(p) = 1010.15$.



[0, 150] by [0, 1600]

59. Using the remainder theorem, the remainder is $(-1)^{40} - 3 = -2$.

60. Using the remainder theorem, the remainder is $1^{63} - 17 = -16$.

61. **(a)** Lower Bound:

−5|	1	2	−11	−13	38
		−5	15	−20	165
	1	−3	4	−33	203