

K E

IF $U = \frac{1}{2}(\text{LOOP})^2$, THEN $DU = (?)$.

Choose a function to substitute for u.

1) $\int 2x(x^2 + 1)^5 dx$	2) $\int \frac{2x}{x^2 + 1} dx$	3) $\int 2x\sqrt{x^2 + 1} dx$	4) $\int 3x^2(x^3 + 1)^5 dx$
5) $\int \sin^3(x)\cos(x)dx$	6) $\int \frac{\cos(x)}{\sin(x)} dx$	7) $-\int \sin(x)e^{\cos(x)} dx$	8) $-\int \frac{\sin(x)}{\cos^2(x)} dx$
9) $\int \tan^3(x)\sec^2(x)dx$		10) $\int \cos(x)\sqrt{1+\sin(x)} dx$	

Let $u = (?)$.

A. $U = \sin^3(x)$	#10 D. $U = 1 + \sin(x)$	#7,8 L. $U = \cos(x)$	M. $U = x^2$	#1,2,3 O. $U = x^2 + 1$
#5,6 P. $U = \sin(x)$	R. $U = \tan^3(x)$	S. $U = e^x$	#9 T. $U = \tan(x)$	#4 U. $U = x^3 + 1$

L	O	O	P	
8	1	1	6	

D	(L	O	O	P)
10	9	7	2	3	5	4

WHAT POLITICAL MOVEMENT AIMS TO PREVENT THE TEACHING OF CALCULUS IN HIGH SCHOOLS?

For the indefinite integrals 1) – 10) above, 1a) – 10a) $du = (?)$.

A. $du = 3x^2 dx$	B. $du = \tan(x) dx$	E. $du = 2x dx$	I. $du = -\sin(x) dx$
K. $du = \sec(x) dx$	M. $du = \sec^2(x) dx$	P. $du = \sin(x) dx$	R. $du = \tan^3(x) dx$
S. $du = e^x dx$	T. $du = \cos(x) dx$	U. $du = -\cos(x) dx$	V. $du = (x^3 + 1) dx$

For the indefinite integrals 1) – 10) above, 1b) – 10b) give the indefinite integral.

A. $\frac{1}{6}(x^2 + 1)^6 + k$	D. $\frac{1}{6}(x^3 + 1)^6 + k$	E. $\frac{2}{3}(x^2 + 1)^{3/2} + k$	H. $\frac{1}{4}\sin^4(x) + k$
M. $\frac{1}{4}\tan^4(x) + k$	N. $-\sec(x) + k$	O. $e^{\cos(x)} + k$	P. $\sec(x) + k$
P. $\sec^3(x) + k$	R. $\frac{2}{3}(\sin x + 1)^{3/2} + k$	T. $\ln(x^2 + 1) + k$	V. $\ln \sin(x) + k$

T	h	e
5a	5b	1a

A	n	d	(
4a	8b	10a	7a

d	e	r	t	i	s	u	v	w
4b	3a	10b	8a	6b	1b	2b	8a	6b

M	O	V	O	I	O	V	x
9a	7b	6b	2a	9b	3b	8b	6a

If $U = \frac{1}{2}(\text{LOOP})^2$, ... puzzle

1. $\int 2x(x^2+1)^5 dx$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x \quad du = 2x dx$$

$$\int u^5 du = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(x^2+1)^6 + C}$$

2. $\int \frac{2x}{x^2+1} dx$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x \quad du = 2x dx$$

$$\int \frac{du}{u} = \ln|u| + C = \boxed{\ln(x^2+1) + C}$$

3. $\int 2x\sqrt{x^2+1} dx$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x \quad du = 2x dx$$

$$\int \sqrt{u} du = \int u^{1/2} du = \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{3}(x^2+1)^{3/2} + C}$$

4. $\int 3x^2(x^3+1)^5 dx$

$$u = x^3 + 1$$

$$\frac{du}{dx} = 3x^2 \quad du = 3x^2 dx$$

$$\int u^5 du = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(x^3+1)^6 + C}$$

5. $\int \sin^3(x) \cos x dx$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x \quad du = \cos x dx$$

$$\int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}(\sin x)^4 + C = \boxed{\frac{1}{4}\sin^4 x + C}$$

$$6. \int \frac{\cos x}{\sin x} dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x \quad du = \cos x dx$$

$$\int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sin x| + C}$$

$$7. - \int \sin x e^{\cos x} dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x \quad du = -\sin x dx$$

$$\int e^u du = e^u + C = \boxed{e^{\cos x} + C}$$

$$8. - \int \frac{\sin x}{\cos^2 x} dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x \quad du = -\sin x dx$$

$$\int \frac{du}{u^2} = \int u^{-2} du = -u^{-1} + C = \frac{-1}{\cos x} + C \\ = \boxed{-\sec x + C}$$

$$9. \int \tan^3 x \sec^2 x dx$$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x \quad du = \sec^2 x dx$$

$$\int u^3 du = \frac{1}{4} u^4 + C = \frac{1}{4} (\tan x)^4 + C = \boxed{\frac{1}{4} \tan^4 x}$$

$$10. \int \cos x \sqrt{1+\sin x} dx$$

$$u = 1 + \sin x$$

$$\frac{du}{dx} = \cos x \quad du = \cos x dx$$

$$\int \sqrt{u} du = \int u^{1/2} du = ?$$

$$\boxed{\frac{2}{3} (1 + \sin x)^{3/2}}$$

Integration Practice Worksheet

(u substitution)

$$1. u = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x dx$$

$$\int u du = \frac{1}{2} u^2 + C$$

$$\frac{1}{2} (\tan x)^2 + C$$

or

$$\boxed{\frac{1}{2} \tan^2 x + C}$$

$$2. u = 1 + 3x + x^3$$

$$\frac{du}{dx} = 3 + 3x^2$$

$$du = (3 + 3x^2) dx$$

$$\frac{1}{3} du = (1 + x^2) dx$$

$$\frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

$$\frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} u^{3/2} + C$$

$$\boxed{\frac{2}{9} (1 + 3x + x^3)^{3/2} + C}$$

$$3. u = 5 + x^3$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{1}{3} \int u^{-1/2} du$$

$$\frac{1}{3} \cdot 2u^{1/2} + C = \frac{2}{3} u^{1/2} + C$$

$$\boxed{\frac{2}{3} \sqrt{5+x^3} + C}$$

$$4. u = \sin x^2$$

$$\frac{du}{dx} = (\cos x^2)(2x)$$

$$\frac{1}{2} du = x \cos x^2 dx$$

$$\frac{1}{2} \int u^3 du$$

$$\frac{1}{2} \cdot \frac{1}{4} u^4 + C = \frac{1}{8} u^4 + C$$

$$\frac{1}{8} (\sin x^2)^4 + C$$

or

$$\boxed{\frac{1}{8} \sin^4 x^2 + C}$$

$$5. u = \sqrt{t}$$

$$\frac{du}{dt} = \frac{1}{2} t^{-1/2}$$

$$2 du = \frac{dt}{\sqrt{t}}$$

$$2 \int e^u du = 2e^u + C$$

$$2e^{\sqrt{t}} + C \Big|_1^4$$

$$2e^2 + C - (2e^1 + C)$$

$$\boxed{2e^2 - 2e}$$

$$\boxed{-e}$$

$$6. u = \sin t$$

$$\frac{du}{dt} = \cos t$$

$$\begin{matrix} \text{U} \\ \text{O} \\ \text{I} \end{matrix}$$

$$du = \cos t dt$$

$$\int u^5 du = \frac{1}{6} u^6 + C$$

$$\begin{matrix} \text{U} \\ \text{O} \\ \text{I} \end{matrix}$$

$$\frac{1}{6} (\sin t)^6 + C \Big|_0^{\pi/2} = \frac{1}{6} (\sin \frac{\pi}{2})^6 + C - \frac{1}{6} (\sin 0)^6 + C$$

$$7. u = 2\theta$$

$$\frac{du}{d\theta} = 2$$

$$\frac{1}{2} du = d\theta$$

$$\frac{1}{2} \int \sec u \tan u du$$

$$\frac{1}{2} \sec u + C$$

$$\boxed{\frac{1}{2} \sec(2\theta) + C}$$

$$8. u = \cos\theta$$

$$\frac{du}{d\theta} = -\sin\theta$$

$$-1 du = \sin\theta d\theta$$

$$-\int u^4 du$$

$$-\frac{1}{5} u^5 + C$$

$$-\frac{1}{5} (\cos\theta)^5 + C$$

or

$$\boxed{-\frac{1}{5} \cos^5\theta + C}$$