AB Calculus Differentiation

In each of the following a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

a. $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ b. $\frac{\sec^2(xy) - y}{x}$ c. $\cos^2(xy)$

d. $\frac{\cos^2(xy)}{x}$ e. $\frac{\cos^2(xy) - y}{x}$

2. $y = \frac{2-x}{3x+1}$

(a) $-\frac{7}{(3x+1)^2}$ (b) $\frac{6x-5}{(3x+1)^2}$ (c) $-\frac{9}{(3x+1)^2}$ (d) $\frac{7}{(3x+1)^2}$ (e) $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(a) $\frac{1}{2\sqrt{3-2x}}$ (b) $-\frac{1}{\sqrt{3-2x}}$ (c) $-\frac{(3-2x)^{\frac{3}{2}}}{3}$ (d) $\frac{-1}{3-2x}$ (e) $\frac{2}{3}(3-2x)^{\frac{3}{2}}$

4. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

a. $\frac{3x-3}{\sqrt{2x-3}}$ b. $\frac{x}{\sqrt{2x-3}}$ c. $\frac{1}{\sqrt{2x-3}}$ d. $\frac{-x+3}{\sqrt{2x-3}}$ e. $\frac{5x-6}{2\sqrt{2x-3}}$

5. $y = 3x^{\frac{2}{3}} - 4x^{\frac{1}{2}} - 2$

(a) $2x^{\frac{1}{3}} - 2x^{\frac{-1}{2}}$ (b) $3x^{\frac{-1}{3}} - 2x^{\frac{-1}{2}}$ (c) $\frac{9}{5}x^{\frac{5}{3}} - 8x^{\frac{3}{2}}$ (d) $\frac{2}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{2}}} - 2$

(e) $2x^{\frac{-1}{3}} - 2x^{\frac{-1}{2}}$

6. $y=\sqrt{x^2+2x-1}$

(a) $\frac{x+1}{y}$ (b) $4y(x+1)$ (c) $\frac{1}{2\sqrt{x^2+2x-1}}$ (d) $-\frac{x+1}{(x^2+2x-1)^{\frac{3}{2}}}$

e) none of these

7. $y=\cos(x^2)$

(a) $2x\sin x^2$ (b) $-\sin x^2$ (c) $-2\sin x\cos x$ (d) $-2x\sin x^2$ (e) $\sin 2x$

8. $y=\frac{1}{2\sin 2x}$

(a) $-\csc 2x\cot 2x$ (b) $\frac{1}{4\cos 2x}$ (c) $-4\csc 2x\cot 2x$ (d) $\frac{\cos 2x}{2\sqrt{\sin 2x}}$

(e) $-\csc^2 2x$

9. $y=\frac{1+x^2}{1-x^2}$

(a) $-\frac{4x}{(1-x^2)^2}$ (b) $\frac{4x}{(1-x^2)^2}$ (c) $-\frac{4x^3}{(1-x^2)^2}$ (d) $\frac{2x}{(1-x^2)}$ (e) $\frac{4}{(1-x^2)}$

10. $x^3-xy+y^3=1$

(a) $\frac{3x^2}{x-3y^2}$ (b) $\frac{3x^2-1}{1-3y^2}$ (c) $\frac{y-3x^2}{3y^2-x}$ (d) $\frac{3x^2+3y^2-y}{x}$

(e) $\frac{3x^2+3y^2}{x}$

11. $\sin x-\cos y-2=0$

(a) $-\cot x$ (b) $-\cot y$ (c) $\frac{\cos x}{\sin y}$ (d) $-\csc y\cos x$

(e) $\frac{2-\cos x}{\sin y}$

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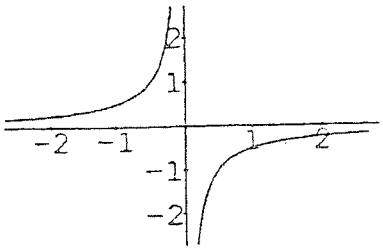
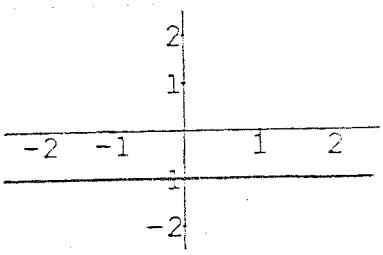
Match the five functions a–e given below with their derivatives (i)–(v). Explain your reasoning.

a

iv

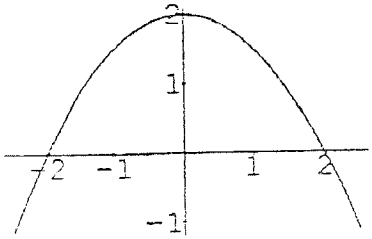


(i)



b

ii



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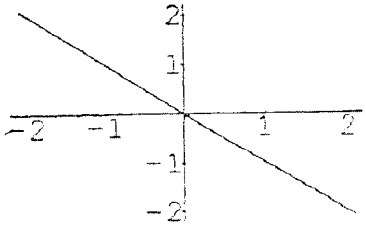
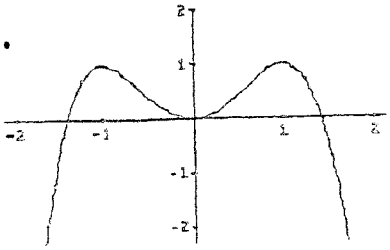
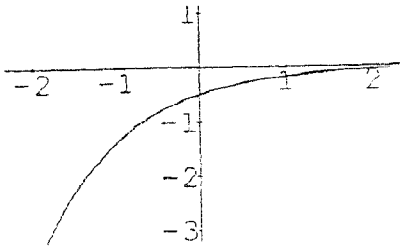


c

v.

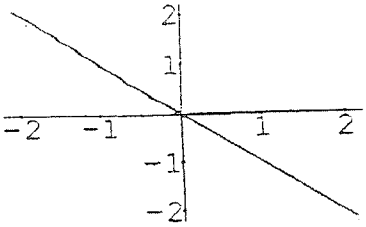


(iii)

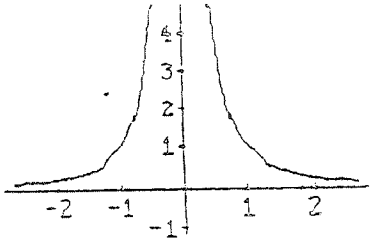


d

i

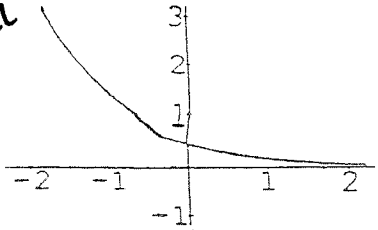


(iv)

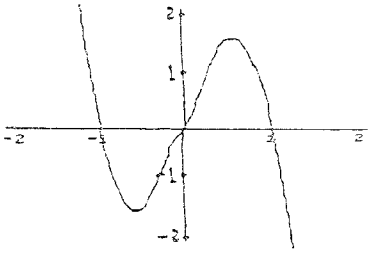


e

iii



(v)



AB Calculus Differentiation

E 1. $\tan(xy) = x$

$$\sec^2(xy)\left[x \cdot \frac{dy}{dx} + y(1)\right] = 1$$

$$x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = 1$$

$$x\sec^2(xy)\frac{dy}{dx} = 1 - y\sec^2(xy)$$

$$\frac{dy}{dx} = \frac{1 - y\sec^2(xy)}{x\sec^2(xy)} = \frac{1}{x\sec^2 xy} - \frac{y\cancel{\sec^2(xy)}}{x\cancel{\sec^2(xy)}}$$

$$= \frac{\cos^2(xy)}{x} - \frac{y}{x} = \frac{\cos^2(xy) - y}{x}$$

A 2. $y = \dfrac{2-x}{3x+1}$

$$y' = \frac{(3x+1)(-1) - (2-x)(3)}{(3x+1)^2} = \frac{-3x-1-6+3x}{(3x+1)^2} = \frac{-7}{(3x+1)^2}$$

B 3. $y = \sqrt{3-2x}$

$$y' = \frac{1}{2}(3-2x)^{-1/2}(-2) = \frac{-1}{\sqrt{3-2x}}$$

A 4. $f(x) = x\sqrt{2x-3}$

$$f'(x) = x \cdot \frac{1}{2}(2x-3)^{-1/2}(2) + \sqrt{2x-3}\,(1)$$

$$= \frac{x}{\sqrt{2x-3}} + \sqrt{2x-3} = \frac{x+2x-3}{\sqrt{2x-3}} = \frac{3x-3}{\sqrt{2x-3}}$$

E 5. $y = 3x^{2/3} - 4x^{1/2} - 2$

$$y' = 2x^{-1/3} - 2x^{-1/2}$$

E 6. $y = \sqrt{x^2+2x-1}$

$$y' = \frac{1}{2}(x^2+2x-1)^{-1/2}(2x+2) = \frac{x+1}{\sqrt{x^2+2x-1}}$$

D 7. $y = \cos(x^2)$

$y' = -\sin(x^2) \cdot (2x)$

A 8. $y = \dfrac{1}{2\sin 2x} = (2\sin 2x)^{-1}$

$y' = -1(2\sin 2x)^{-2}(2\cos 2x) \cdot 2 = \dfrac{-4\cos 2x}{4\sin^2 2x}$

$= \dfrac{-\cos 2x}{\sin^2 2x} = -\dfrac{\cos 2x}{\sin 2x} \cdot \dfrac{1}{\sin 2x} = -\cot 2x \csc 2x$

B 9. $y = \dfrac{1+x^2}{1-x^2}$

$y' = \dfrac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}$

$= \dfrac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} = \dfrac{4x}{(1-x^2)^2}$

C 10. $x^3 - xy + y^3 = 1$

$3x^2 - \left[x\dfrac{dy}{dx} + y(1)\right] + 3y^2\dfrac{dy}{dx} = 0$

$-x\dfrac{dy}{dx} + 3y^2\dfrac{dy}{dx} = y - 3x^2$

$\dfrac{dy}{dx} = \dfrac{y - 3x^2}{3y^2 - x}$

D 11. $\sin x - \cos y - 2 = 0$

$\cos x - (-\sin y)\dfrac{dy}{dx} - 0 = 0$

$\sin y \dfrac{dy}{dx} = -\cos x$

$\dfrac{dy}{dx} = -\dfrac{\cos x}{\sin y} = -\cos x \cdot \dfrac{1}{\sin y} = -\cos x \csc y$