

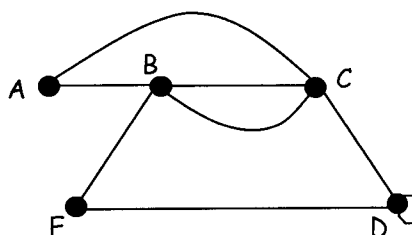
- 1) An election is being held to choose the president of the Math Anxiety Club. There are four candidates: Amy, Brad, Chris, and Dinah (A, B, C, and D for short). Each of the 71 members of the club is asked to submit a ballot indicating his/her first, second, third, and fourth choices. The 71 ballots are summarized in the following table:

First choice	B	D	C	B	A
Second choice	D	C	B	C	D
Third choice	C	B	D	D	B
Fourth choice	A	A	A	A	C
Number of voters	19	10	15	2	25

- a. Find the winner of the election using the plurality method. A
- b. Find the winner of the election using the run-off method. B
- c. Find the winner of the election using the sequential run-off method. C
- d. Determine the winner of the election using a Borda count. D
- e. If possible, determine the Condorcet winner of the election. B
- f. Suppose that this election is conducted by the approval method and that each voter approves of the first two choices on his/her preference schedule. Determine the approval winner. D

- 2) Given the graph below find the following:

- a. What type of graph is it (i.e. directed, directed multigraph, pseudograph, multigraph, or simple)?
- b. # of vertices 5
- c. # of edges 8
- d. Find the valence of each vertex:



vertex	valence/degree
A	2
B	4
C	4
D	4
E	2

Vertex	adjacent to
A	B, C
B	A, C, E
C	A, B, D
D	C, D, E
E	B, D

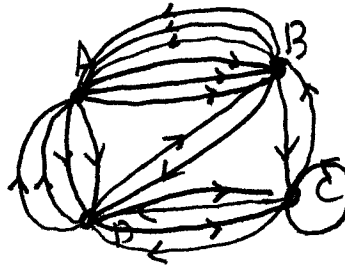
0	1	1	0	0
1	0	2	0	1
1	2	0	1	0
0	0	1	1	1
0	1	0	1	0

- e. Represent the graph with an adjacency list.
- f. Represent the graph with an adjacency matrix.
- g. Does the graph have an Euler path, an Euler circuit, or neither? Explain.
each vertex has an even degree
- h. Find a path from vertex A to vertex C with a length of 4.
(answers vary) Sample answers: ABCBC, ABEDC
- i. Does the graph have a Hamilton path, a Hamilton circuit, or neither? Explain.

sample answer: BACDEB visits each vertex once & returns to the starting vertex

- b) Given the adjacency matrix, construct a directed graph. Label the vertices with A, B, C, & D.

	A	B	C	D
A	0	3	0	2
B	3	0	1	1
C	0	1	1	2
D	2	1	2	0



- 4) In the absence of a will, Andrew, Megan, and Olivia must fairly divide their uncle's estate consisting of a beach cabin and a boat. The group agrees to use the method of sealed bids. The bids are shown in the chart below.

	Andrew	Megan	Olivia
Boat	\$16,000	<u>\$20,000</u>	\$12,000
Cabin	\$80,000	\$70,000	<u>\$90,000</u>

$$A: 0 - 32000 = -32000$$

$$M: 20000 - 30000 = -10000$$

$$O: 90000 - 34000 = 56000$$

- a. Find each person's fair share. \div TB by 3

total bids 96 000 90 000 102 000

Andrew: 32000 Megan: 30000 Olivia: 34000

$$\begin{array}{r} 56000 \\ -42000 \\ \hline 14000 \text{ surplus} \\ \div 3 = 4666.67 \end{array}$$

- b. Who receives the items? Boat: Megan Cabin: Olivia

- c. After the initial calculations, does anyone need to pay money into the estate? If so, who and how much? Olivia \rightarrow \$56000

- d. After each person is awarded the equivalent of his or her fair share, how much surplus cash remains? \$14000

- e. After all the items and the surplus cash have been awarded, what is the final settlement for each of the three heirs?

Andrew: receives \$36666.67

Megan: gets boat and \$14666.67

Olivia: gets cabin; pays \$51,333.33

- 5) Rex Hospital has 10 doctors to be apportioned to three clinics. The hospital board decides to apportion the doctors based on the average weekly patient load for each clinic, given in the following table.

- A. Find the standard divisor. $SD = \frac{500}{10} = 50$
- B. Find each clinic's standard quota.
- C. Find each clinic's lower quota and upper quota.
- D. Use each of the 5 apportionment methods we learned in class to apportion the doctors to the clinics.

	Gm
A	2.44949
B	3.46410
C	4.47236

Clinic	Average Weekly Patient Load	Standard quota	Lower quota	Upper quota	Hamilton	Jefferson MD=42	Webster MD=48	Huntington-Hill MD=48.5
A	119	2.38	2	3	3	2	2	3
B	165	3.3	3	4	3	3	3	3
C	216	4.32	4	5	4	5	5	4

SDU

6) Consider the weighted voting situation in which voters A, B, and C have 18, 10, and 4 votes, respectively, and 14 votes are needed to pass an issue: [14: 18, 10, 4].

a. Complete the chart below.

coalition	weight of the coalition	Is it a winning coalition? (yes/no)	List of the critical voters of the winning coalitions
A	18	yes	A
B	10	no	—
C	4	no	—
AB	28	yes	A
AC	22	yes	A
BC	14	yes	B, C
ABC	32	yes	none

b. Find the Banzhof power index for each voter.

$$A = \frac{3}{5}$$

$$B = \frac{1}{5}$$

$$C = \frac{1}{5}$$

c. Find the Shapley-Shubik power index for each voter.

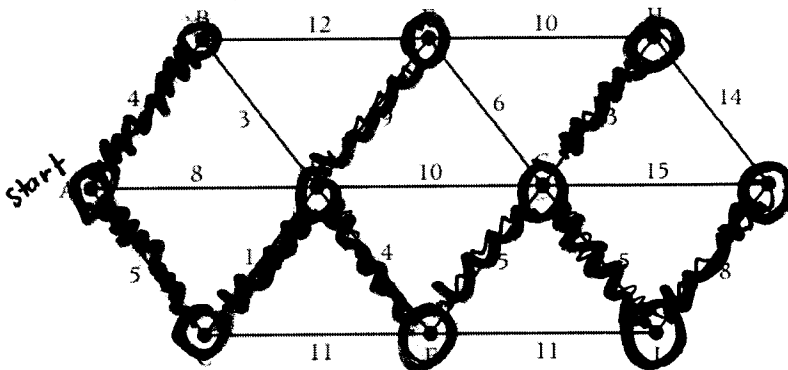
$\underline{A}BC$ $B\underline{A}C$ $C\underline{A}B$
 $\underline{A}CB$ $B\underline{C}A$ $C\underline{B}A$

$$A = \frac{4}{6} = \frac{2}{3}$$

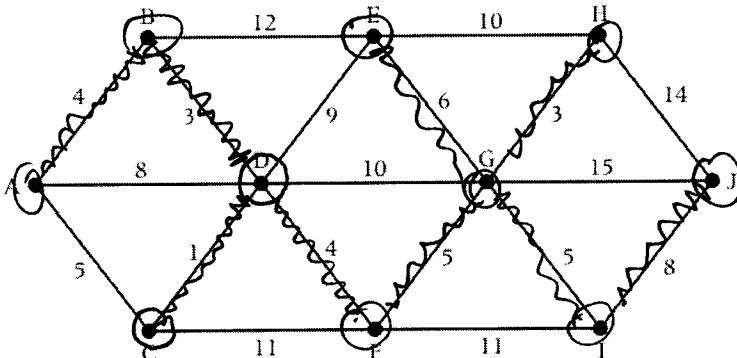
$$B = \frac{1}{6}$$

$$C = \frac{1}{6}$$

6) For the graph below, determine the shortest path from vertex A to each of the other vertices.

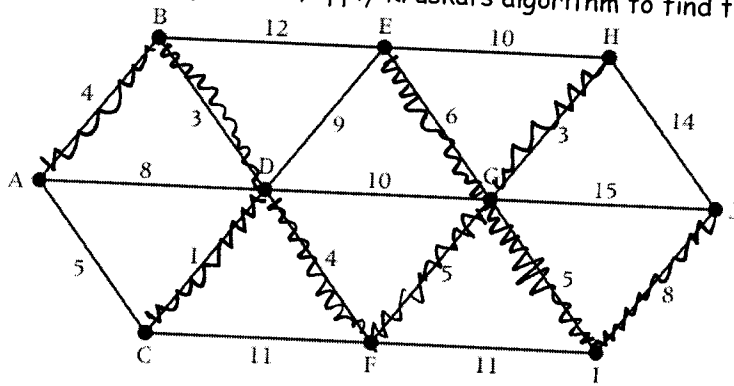


7) For the graph below, apply Prim's algorithm to find the minimum spanning tree and its weight.



$$\text{weight} = 39$$

For the graph below, apply Kruskal's algorithm to find the minimum spanning tree and its weight.

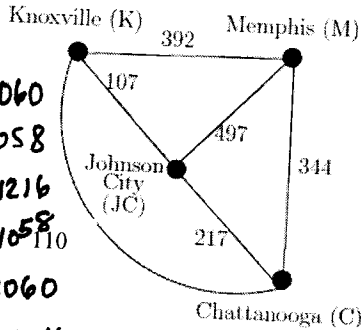


10 vertices \Rightarrow shade 9 edges

weight = 39

9) Farmer Tucker lives in Johnson City and must deliver produce to other cities in Tennessee. Apply the Brute Force method and the Nearest Neighbor method to determine the best route for Farmer Tucker to take to minimize the distance driven.

Brute Force



- Jc K M C Jc = 1060
- Jc K C M Jc = 1058
- Jc M K C Jc = 1216
- Jc M C K Jc = 1058
- Jc C M K Jc = 1060
- Jc C K M Jc = 1216

Brute Force solution(s) Jc K C M Jc and Jc M C K Jc

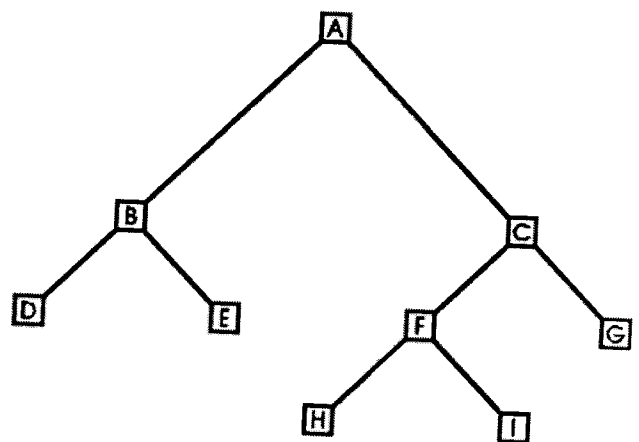
weight = 1058

Nearest Neighbor solution(s) Jc K C M Jc

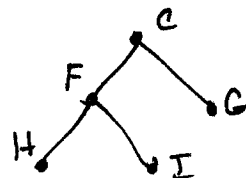
weight = 1058

10) For the given tree, find:

- A a. the root
- A, B, C, F b. the internal vertices
- D, E, G, H, I c. the leaves
- F, G d. the child(ren) of C
- E e. the sibling(s) of D
- F f. the parent of H
- C, A g. the ancestors of G
- F, G, H, I h. the descendants of C
- 2 i. the level of F
- 3 j. the height of the tree

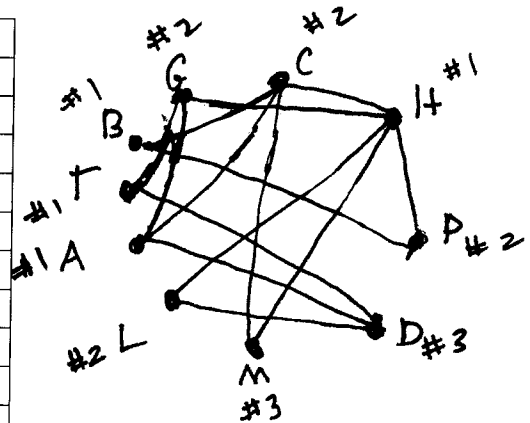


11) For the tree in problem #10, sketch the subtree rooted at C.



Suppose that 10 argumentative relatives are attending a wedding reception. Some of these family members fight constantly, so they cannot be seated at the same table. We want to use the smallest number of tables. All incompatible pairs of relatives are given below.

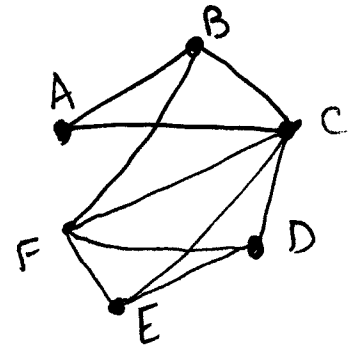
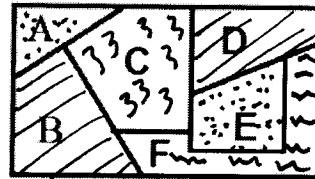
relative	cannot be seated with:
Gina	Herman, Anne, Tucker
Chad	Herman, Anne, Misty, Beth
Herman	Gina, Pam, Misty, Lilly
Pam	Herman, Beth
Daisy	Anne, Tucker, Lilly
Misty	Chad, Herman
Lilly	Herman, Daisy
Anne	Gina, Chad, Daisy
Tucker	Gina, Daisy
Beth	Chad, Pam



- Represent the problem as a graph in the space beside the table.
- Find the chromatic number. 3
- What is the least number of tables required? 3
- List the relatives who can be seated together. (Use only the minimum number of tables)
< sample answer > #1 Beth, Herman, Anne, Tucker
 #2 Gina, Chad, Pam, Lilly
 #3 Daisy, Misty

13. For the given map:

- Model it with a graph.
- chromatic number = 4
- Color the map using the minimum number of colors necessary.



14. Sketch a binary search tree in alphabetical order for the following words:
 Mace, Nick, Grace, Riley, Megan, Madison, Moira

