Pre-Calculus Name: ____________________

Chapter 6 Review --- Vectors, Polars, & Parametrics

In the following exercises, eliminate the parameter and describe the graph of the function. (Be sure you take domain and range of the parametric into account)

1. $x = t$ & $y = 2t - 1$

$\boxed{y = 2x - 1}$

linear

2. $x = 4\cos^2\theta$ & $y = 2\sin\theta$

$\frac{x}{4} = \cos^2\theta$ $\frac{y}{2} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$(\frac{x}{4}) + (\frac{y}{2})^2 = 1$

$\frac{x}{4} + \frac{y^2}{4} = 1$

parabola

$x + y^2 = 4$

$y^2 = -x + 4$

$y = \pm\sqrt{-x+4}$

3. $x = 4t - 1$ & $y = 2t + 3$

$t = \frac{x+1}{4}$ $y = 2(\frac{x+1}{4}) + 3$

$\frac{2x+2}{4} + 3$

linear $\boxed{y = \frac{1}{2}x + \frac{7}{2}}$

4. $x = t + 3$ & $y = t^2$

$t = x - 3$ $y = (x-3)^2$

$(x-3)(x-3)$

quadratic

$\boxed{y = x^2 - 6x + 9}$

5. $x = \sqrt[3]{t}$ & $y = 3 - t^2$

$x^3 = t$ $y = 3 - (x^3)^2$

$\boxed{y = -x^6 + 3}$

polynomial

6. $x = t^2 - 1$ & $y = t^2 + 1$

$t = \pm\sqrt{x+1}$ $y = (\pm\sqrt{x+1})^2 + 1$

$x + 1 + 1$

linear $\boxed{y = x + 2}$

7. $x = t - 2$ & $y = \dfrac{t}{t-2}$

$t = x + 2$

$y = \frac{x+2}{(x+2) - 2}$ $\boxed{y = \frac{x+2}{x}}$

rational function

★ **9.** $x = |t - 3|$ & $y = t + 3$

$x = t - 3$ $-x = t - 3$

$x + 3 = t$ $-x + 3 = t$

$t = \pm x + 3$

$y = (\pm x + 3) + 3$

$\boxed{y = x + 6 \quad y = -x + 6}$

linear

10. $x = \sec^2\theta$ & $y = \tan^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + y = x$

$\boxed{y = x - 1}$ linear

11. $x = \cos\theta$ & $y = 4\sin\theta$

$\cos^2\theta + \sin^2\theta = 1$ $\frac{y}{4} = \sin\theta$

$16(x^2 + \frac{y^2}{16} = 1)$

$16x^2 + y^2 = 16$ $y^2 = -16(x^2 - 1)$

$y^2 = -16x^2 + 16$ $\boxed{y = \pm\sqrt{-16(x^2-1)}}$

12. $x = e^t$ & $y = e^{-t}$

$\ln x = t$ $y = e^{-\ln x}$

$\ln y = -\ln x$

linear $\boxed{y = -x}$

13. $x = t^5$ & $y = 5\ln t$

$\frac{y}{5} = \ln t$

$e^{\frac{y}{5}} = t$

$x = (e^{\frac{y}{5}})^5$

$x = e^y$

$\boxed{y = \ln x}$ logarithmic

14. A dart is thrown upward from 6 ft. high with an initial velocity of 18 feet/sec at an angle of elevation of 41° θ

v_0

a. Write a parameterization describing the position of the dart at time t.

$x(t) =$ $18\cos(41°)t$ $y(t) =$ $-16t^2 + 18\sin(41°)t + 6$

$\boxed{\text{MODE}}$ par + degree

b. Approximately how long will it take for the dart to hit the ground?

$t \approx 1.1$ seconds

$\boxed{\text{WINDOW}}$

t-min: 0 X-min: -10 Y-min: -10

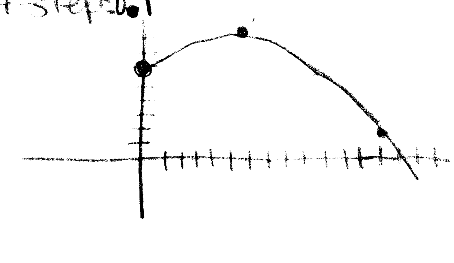
t max: 20 X-max: 20 Y-max: 20

t step: 0.1

c. Find the approximate maximum height of the dart. ≈ 8 ft high at .4 seconds (5.43, 8.16)

d. How long will it take for the dart to reach maximum height?

≈ 0.5 seconds



15. An arrow is shot from a platform 20 feet off the ground with an initial velocity of 150 feet/sec at an angle of elevation of 23°.

a. Write a parameterization describing the position of the arrow at time t.

MODE par, degree

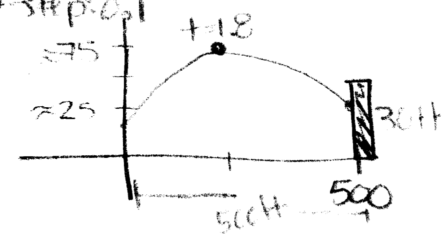
$x(t) =$ 150cos 23° t $y(t) =$ −16t² +150 sin 23° t +20

t min: 0 x-min: -5 y-min: -10
t max: 20 x-max: 500 y-max: 100
t step: 0.1

b. Find the approximate maximum height of the arrow. ≈ 75 ft

c. Approximately how long will it take for the arrow to reach maximum height?
≈ 1.8 seconds

d. There is a wall 30 feet high 500 feet from the archer. Will the arrow hit it? If so, how long will it take to hit it? It hits the wall at about 3.6 seconds



16. A golfer hits a ball with an initial velocity of 90 mph at angle of elevation of 64°. Same window as #15, change y max: 150

a, Write a parametric equation that describes the position of the ball at time t.

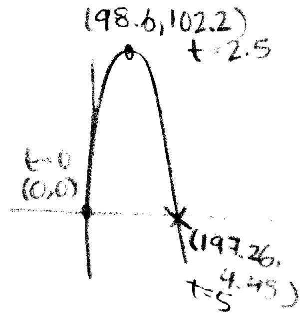
$x(t) =$ 90cos 64° t $y(t) =$ −16t² + 90 sin 64° t

b. Approximately how long will it take for the ball to hit the ground? ≈ 5 seconds

c. Find the approximate maximum height of the ball. ≈ 102 ft

d, The green is 150 yards away. Will the ball reach the green? Explain.

Yes, it hits the ground at ≈ 197 ft out.



17. An NFL kicker at the 33-yard line attempts a field goal. The ball leaves his foot at 69 feet/sec at an angle of elevation of 38°.

a. Write a parametric equation that describes the position of the ball at time t.

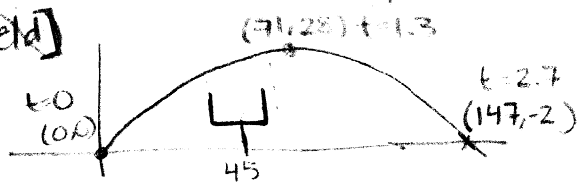
Make window smaller
x-min: -5 y-min: -10
x-max: 200 y-max: 50

$x(t) =$ 69cos 38° t $y(t) =$ −16t² + 69 sin 38° t

b. How high does the ball get above the field?
max height ≈ 28 ft [not necessarily over field]

b **c.** The goal posts are 10 ft high & 45 yards away from him. If the kick is straight, is the field goal good? Explain

Yes, it is good. At x=45 the ball is > 10 ft above ground (≈ 24 ft)



18. Jack and Jill are standing 60 feet apart. At the same time, they each throw a softball from an initial height of two feet towards each other. Jack throws the softball at an initial velocity of 45 ft/sec at an angle of elevation of 44°. Jill throws her ball with an initial velocity of 41 ft/sec with an angle of elevation of 37°.

x-min: -5 y-min: -10
x-max: 100 y-max: 20

a. Write 2 parameterizations describing the position of the balls at time t.
Remember they are throwing the balls toward each other.

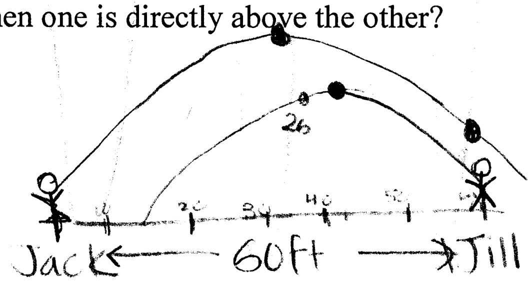
Jack $x_1(t) =$ 45cos 44° t $y_1(t) =$ −16t² + 45 sin 44° t + 2 Jill $x_2(t) =$ 41cos 37° t $y_2(t) =$ −16t² + 41 cos 37° t + 2

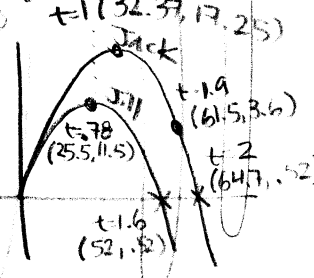
b. Find the heights of each ball when one is directly above the other. 45cos44° t = 41cos37° t, t(45cos44° − 41cos37°) = 0 t = 1 (32.37, 17.25)

c. About how far has each ball traveled when one is directly above the other?

d. When does each ball hit the ground?

Jack's → ≈ 2 seconds
Jill's → ≈ 1.6 seconds





19. Convert the following polar points to rectangular coordinates.

a. $\left(6, \frac{\pi}{2}\right)$ $(6\cos\frac{\pi}{2}, 6\sin\frac{\pi}{2})$
$6(0), 6(1)$
$\boxed{(0,6)}$

b. $\left(-1, \frac{7\pi}{4}\right)$ $(-1\cos\frac{7\pi}{4}, -1\sin\frac{7\pi}{4})$
$-1(\frac{\sqrt{2}}{2}), -1(-\frac{\sqrt{2}}{2})$
$\boxed{\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$
$7\pi/4$

c. $\left(-4, -\frac{\pi}{3}\right)$ $(-4\cos-\frac{\pi}{3}, -4\sin-\frac{\pi}{3})$
$-4(\frac{1}{2}), -4(-\frac{\sqrt{3}}{2})$
$\boxed{(-2, 2\sqrt{3})}$
$-\frac{\pi}{3}$

d. (3, 120°) $(3\cos 120°, 3\sin 120°)$
$3(-\frac{1}{2}), 3(\frac{\sqrt{3}}{2})$
$\boxed{\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)}$

e. (8, 210°) $(8\cos 210°, 8\sin 210°)$
$8(-\frac{\sqrt{3}}{2}), 8(-\frac{1}{2})$
$\boxed{(-4\sqrt{3}, -4)}$

f. (10, 72°) **Need calc for this one
$(10\cos 72°, 10\sin 72°)$
$\boxed{(3.1, 9.5)}$

20. Convert the following rectangular points to polar coordinates

a. (–3, 3) $\boxed{(3\sqrt{2}, 135°)}$
$r = \sqrt{3^2+(-3)^2} = \sqrt{18} = 3\sqrt{2}$
$\tan^{-1}\left(\frac{3}{-3}\right) = 45°$
$180-45 = \theta = 135°$

b. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\boxed{(1, -60°)}$
$r = \sqrt{(\frac{1}{2})^2 + (-\frac{\sqrt{3}}{2})^2}$
$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$
$\tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = -60°$

c. (0, –4) $\boxed{(4, 270°)}$

d. $\left(-5\sqrt{2}, -5\sqrt{2}\right)$ $\boxed{(10, 225°)}$
$r = \sqrt{(-5\sqrt{2})^2 + (-5\sqrt{2})^2}$
$\sqrt{25\cdot 2 + 25\cdot 2} = \sqrt{100} = 10$
$\tan^{-1}\left(\frac{-5\sqrt{2}}{-5\sqrt{2}}\right) = 45°$
$180 + 45° = 225 = \theta$

e. $(-\sqrt{3}, 1)$ $\boxed{(2, 150°)}$
$r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$ $\theta = 150°$
$\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = 30°$ $180 - 30°$

f. (7, –24) $\boxed{(25, -73.74°)}$
$r = \sqrt{7^2 + (-24)^2} = \sqrt{625} = 25$
$\tan^{-1}\left(\frac{-24}{7}\right) = -73.74°$

21. For each of the following rectangular equations, change it to polar form.

a. $x^2 - y^2 = 4$
$(r\cos\theta)^2 - (r\sin\theta)^2 = 4$
$r^2\cos^2\theta - r^2\sin^2\theta = 4$
$r^2(\cos^2\theta - \sin^2\theta) = 4$
$r^2(\cos 2\theta) = 4$
$\boxed{r = \pm\frac{2}{\sqrt{\cos 2\theta}}}$ $\boxed{r = \sqrt{\frac{4}{\cos 2\theta}}}$

b. $xy = 12$
$r\cos\theta\, r\sin\theta = 12$
$r^2 = \frac{12}{\cos\theta\sin\theta}$
$\boxed{r = \pm\sqrt{\frac{12}{\cos\theta\sin\theta}}}$
or $\boxed{r = \pm\sqrt{12\sec\theta\csc\theta}}$

c. $5x - y = 7$
$5r\cos\theta - r\sin\theta = 7$
$r(5\cos\theta - \sin\theta) = 7$
$\boxed{r = \frac{7}{5\cos\theta - \sin\theta}}$

d. $(x-1)^2 + y^2 = 1$
$(x-1)(x-1)$
$x^2 - 2x + 1 + y^2 = 1$
$x^2 - 2x + y^2 = 0$
$r^2 - 2r\cos\theta = 0$
$\cancel{r}(r - 2\cos\theta) = 0$
$\boxed{r = 2\cos\theta}$

e. $y = x\sqrt{3}$
$\frac{y}{x} = \sqrt{3}$
$\boxed{\tan\theta = \sqrt{3}}$

f. $x^2 + y^2 + 4x = 0$
$r^2 + 4r\cos\theta = 0$
$\cancel{r}(r + 4\cos\theta) = 0$
$\boxed{r = -4\cos\theta}$

22. For each of the following polar equations, change it to rectangular form.

$\tan\theta = \frac{y}{x}$

a. $(r)^2 = (4)^2$
$r^2 = 16$
$\boxed{x^2 + y^2 = 16}$

b. $\tan^2\theta = 9$
$\left(\frac{y}{x}\right)^2 = 9$
$\frac{y^2}{x^2} = 9$
$y^2 = 9x^2$
$y = \pm\sqrt{9x^2}$
$\boxed{y = \pm 3x}$

c. $r = 8\csc\theta$
$\frac{1}{r}\left(r = \frac{8}{\sin\theta}\right)$
$1 = \frac{8}{r\sin\theta}$
$r\sin\theta = 8$
$\boxed{y = 8}$

d. $r(r = 8\cos\theta)$
$r^2 = 8r\cos\theta$
$x^2 + y^2 = 8x$
$x^2 - 8x + y^2 = 0$
$\boxed{(x-4)^2 + y^2 = 16}$

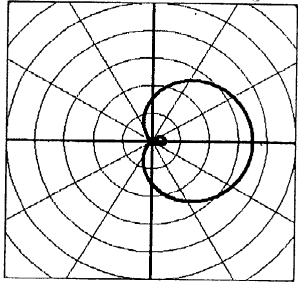
e. $r = \dfrac{5}{2\sin\theta - \cos\theta}$
$2r\sin\theta - r\cos\theta = 5$
$2y - x = 5$
$2y = x + 5$
$\boxed{y = \frac{1}{2}x + \frac{5}{2}}$

f. $r = \dfrac{1}{1 + \cos\theta}$
$r + r\cos\theta = 1$
$r + x = 1$
$(r)^2 = (-x+1)^2$
$r^2 = (-x+1)(-x+1)$
$x^2 + y^2 = x^2 - 2x + 1$
$y^2 = -2x + 1$
$\boxed{y = \pm\sqrt{-2x+1}}$

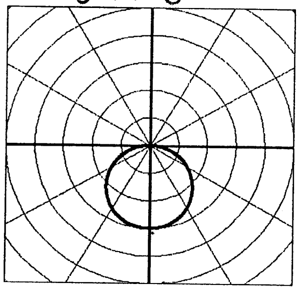
23. Match the polar equations with their graphs below.

K 1) $r = 2.5 + 2.5\sin\theta$	H 2) $r = 3$	M 3) $r = 3.5\sin(3\theta)$	P 4) $r = 4.5\sin(2\theta)$
L 5) $r = 4.5\cos(2\theta)$	A 6) $r = 1.5 + 2\cos\theta$	B 7) $r = -3\sin\theta$	N 8) $r = 2 - \sin\theta$
G 9) $r^2 = 16\sin(2\theta)$	O 10) $r = 4\cos(5\theta)$	E 11) $r = 3.5\cos(3\theta)$	F 12) $r = 2.5 - 2.5\cos\theta$
I 13) $r = 3\cos\theta$	J 14) $r = 1 + 4\sin\theta$	C 15) $r = 4.5\sin(6\theta)$	D 16) $r = \frac{1}{2}\theta$

A. $a \pm b\cos\theta$ $\frac{a}{b} < 1$

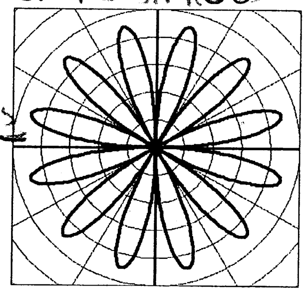


B. $r = -3\sin\theta$

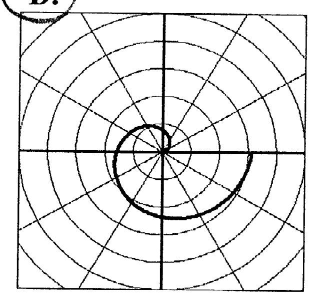


C. 4.5 or cos $r = \sin(6\theta)$

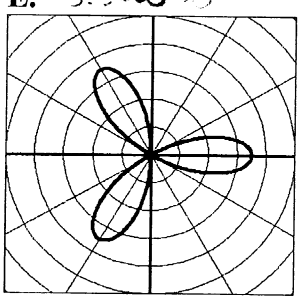
12 petals



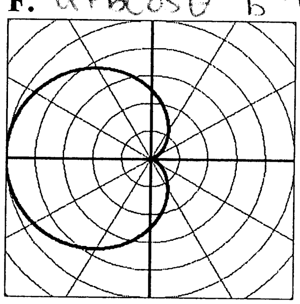
D.



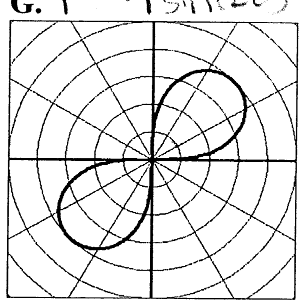
E. $3.5\cos 3\theta$



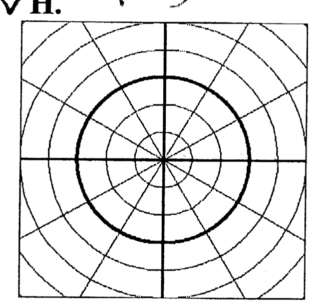
F. $a + b\cos\theta$ $\frac{a}{b} = 1$



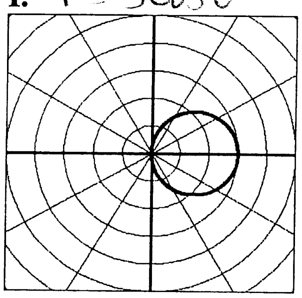
G. $r^2 = 4^2\sin(2\theta)$



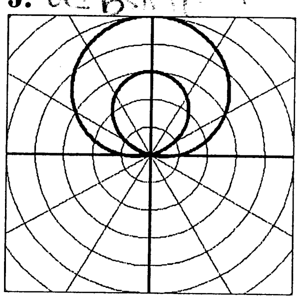
H. $r = 3$



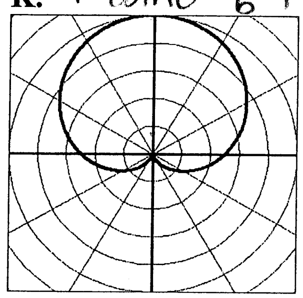
I. $r = 3\cos\theta$



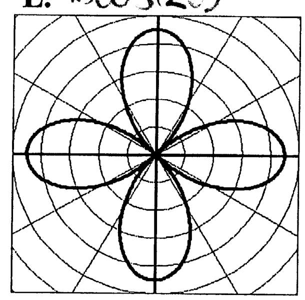
J. $a \pm b\sin\theta$ $\frac{a}{b} < 1$



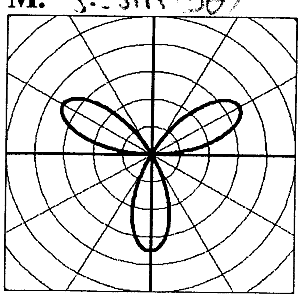
K. $a \pm b\sin\theta$ $\frac{a}{b} = 1$



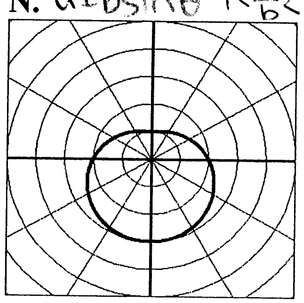
L. $4.5\cos(2\theta)$



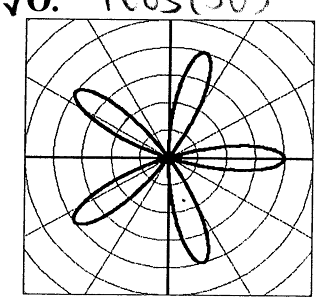
M. $3.5\sin(3\theta)$



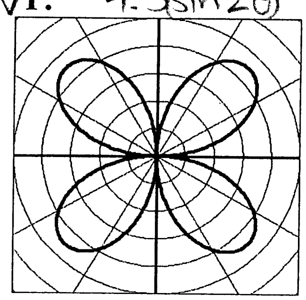
N. $a \pm b\sin\theta$ $1 < \frac{a}{b} < 2$



O. $4\cos(5\theta)$



P. $4.5\sin 2\theta$



25. Find component form and the magnitude of the vector **v** with initial point P and terminal point Q.

a. $P(-3,7), Q(2,-1)$

$\langle 2+3, -1-7\rangle$ $\sqrt{5^2+(-8)^2}$

$\boxed{\mathbf{v} = \langle 5, -8\rangle \quad |\mathbf{v}| = \sqrt{89}}$

b. $P(6,-2), Q(-1,-2)$

$\langle -1-6, -2+2\rangle$ $\sqrt{7^2+0^2}$

$\mathbf{v} = \langle -7, 0\rangle \quad |\mathbf{v}| = 7$

c) $P(-4,3), Q(-5,1)$

$\langle -5+4, 1-3\rangle$ $\sqrt{(-1)^2+2^2}$

$\mathbf{v} = \langle -1, 2\rangle \quad |\mathbf{v}| = \sqrt{5}$

26. Given the vectors $\mathbf{u} = \langle 1,-3\rangle$, $\mathbf{v} = \langle 3,9\rangle$, find the following:

a. u + v

$\boxed{\langle 4, 6\rangle}$

$\langle 1+3, -3+9\rangle$

b. u − v

$\langle -2, -12\rangle$

$\langle 1-3, -3-9\rangle$

c. 8u − 5v

$\langle -7, -69\rangle$

$8\langle 1,-3\rangle - 5\langle 3,9\rangle$

$\langle 8,-24\rangle - \langle 15,45\rangle$

d. u · v

-24

$1\cdot 3 + -3\cdot 9$

$3 + -27$

e. $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\boxed{\left\langle -\frac{4}{5}, \frac{-12}{5}\right\rangle}$

$u\cdot v = -24$ $\quad \frac{-24}{90}\langle 3,9\rangle$

$|v|^2 = (\sqrt{3^2+9^2})^2 = 90$

f. Write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$)

$\vec{u} = \left\langle \frac{-4}{5}, \frac{-12}{5}\right\rangle + \left\langle \frac{9}{5}, \frac{-3}{5}\right\rangle$

$\langle 1,-3\rangle - \left\langle \frac{-4}{5}, \frac{-12}{5}\right\rangle$

g. The angle between **u** and **v**

$143.13°$

$|u| = \sqrt{1^2+(-3)^2}$

$\cos^{-1}\left(\frac{-24}{\sqrt{90}\sqrt{10}}\right) = \theta$

27. Find a unit vector in the direction of the following vectors and show that it has length 1.

a. $\mathbf{v} = \langle 2,-13\rangle$

$\left\langle \frac{2}{\sqrt{173}}, \frac{-13}{\sqrt{173}}\right\rangle$

$\frac{\langle 2,-13\rangle}{\sqrt{173}}$

b. $\mathbf{v} = \langle -9,-7\rangle$

$\left\langle \frac{-9}{\sqrt{130}}, \frac{-7}{\sqrt{130}}\right\rangle$

$\frac{\langle -9,-7\rangle}{\sqrt{130}}$

c. $\mathbf{v} = \langle -14,0\rangle$

$\langle -1, 0\rangle$

$\frac{\langle -14,0\rangle}{\sqrt{196}}$

28. Let **u** be the vector with initial point (13, 5) and terminal point (−12, 5) and let $\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$. Write the following as a linear combination of **i** and **j**. $u = \langle -25, 0\rangle$ $\quad v = \langle 8, 6\rangle$

a. $-2\mathbf{u}$

$-2\langle -25,0\rangle$

$\boxed{\langle 50, 0\rangle}$

b. $\mathbf{u} - 2\mathbf{v}$

$\langle -25,0\rangle - 2\langle 8,6\rangle$

$\langle -25,0\rangle - \langle 16,12\rangle$

$\boxed{\langle -41, -12\rangle}$

c. $\frac{\mathbf{u}}{|\mathbf{v}|}$

$|v| = \sqrt{8^2+6^2} = \sqrt{100} = 10$

$\frac{\langle -25, 0\rangle}{10} = \left\langle \frac{-25}{10}, \frac{0}{10}\right\rangle =$

$\boxed{\left\langle -\frac{5}{2}, 0\right\rangle}$

$\theta = \tan^{-1}\left(\frac{-7}{24}\right)$

29. Write the vector **v** given its magnitude and direction angle.

$\langle 24, -7\rangle$ $\theta = -16.26$

a. $|\mathbf{v}| = 13$ $\theta = 60°$

$\boxed{\left\langle \frac{13}{2}, \frac{13\sqrt{3}}{2} \right\rangle}$

$\langle 13\cos 60°, 13\sin 60°\rangle$

b. $|\mathbf{v}| = 20$ $\theta = 150°$

$\boxed{\langle -10\sqrt{3}, 10\rangle}$

$\langle 20\cos 150°, 20\sin 150°\rangle$

c. $|\mathbf{v}| = 5$ θ = direction of $24\mathbf{i} - 7\mathbf{j}$

$\boxed{\left\langle \frac{24}{5}, \frac{-7}{5} \right\rangle}$

$\langle 5\cos(-16.26), 5\sin(-16.26)\rangle$

from North

30. A plane is flying on a bearing of 320° at 375 mph. A wind is blowing with the bearing 305° at 45mph.

a. Write a vector (in component form) of the velocity produced by the airplane alone.

130°

$\langle 375\cos 130°, 375\sin 130°\rangle$

p = $\langle -241.05, 287.27\rangle$

b. Write a vector (in component form) of the velocity of the wind.

145°

$\langle 45\cos 145°, 45\sin 145°\rangle$

w = $\langle -36.86, 25.81\rangle$

c. Write a vector (in component form) of the actual velocity of the plane.

$\langle 375\cos 130° + 45\cos 145°, 75\sin 130 + 45\sin 145\rangle$ v = $\langle -277.91, 313.08\rangle$

d. Find the actual speed and direction angle (not the bearing) of the plane.

$\sqrt{(-277.91)^2 + (313.08)^2}$ speed

$\tan^{-1}\left(\frac{313.08}{-277.91}\right) = \theta$

$180 - 48$

48.4

speed = 418.6 mph θ = 131.6°

31. Find the vector projection **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle 3, -9\rangle$ & $\mathbf{v} = \langle -1, 7\rangle$

$|v| = \sqrt{(-1)^2 + 7^2} = \sqrt{50}$

$u \cdot v = 3 \cdot -1 + -9 \cdot 7$

$-3 + -63 = -66$

$|v|^2 = 50$

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{33}{25}, \frac{-231}{25} \right\rangle$

$\mathbf{u} = \left\langle \frac{33}{25}, \frac{-231}{25} \right\rangle + \left\langle \frac{42}{25}, \frac{6}{25} \right\rangle$

$\text{proj}_v U = W_1 = \frac{-66}{50}\langle 1, 7\rangle = \left\langle \frac{66}{50}, \frac{-462}{50} \right\rangle$

$U - W_1 = W_2$

$W_2 = \langle 3, -9\rangle - \left\langle \frac{33}{25}, \frac{-231}{25} \right\rangle$