Practice with Dot Products and Angles between Vectors

1) Find the dot product of **u** and **v**. Then determine if it is orthogonal.

a) $u = \langle 3,-2 \rangle$ and $v = \langle -5,1 \rangle$

$3 \cdot -5 + -2 \cdot 1$
$-15 + -2$
-17 no

b) $u = \langle -2,-3 \rangle$ and $v = \langle 9,-6 \rangle$

$-2 \cdot 9 + -3 \cdot -6$
$-18 + 18$
0 yes

c) $u = \langle -3,4 \rangle$ and $v = \langle 3,6 \rangle$

$-3 \cdot 3 + 4 \cdot 6$
$-9 + 24$
15 no

d) $u = \langle 2,7 \rangle$ and $v = \langle -14,4 \rangle$

$2 \cdot -14 + 7 \cdot 4$
$-28 + 28$
0 yes

2) Find the magnitude of:

a) $c = \langle -1,-7 \rangle$

$\sqrt{(-1)^2 + (-7)^2}$
$\sqrt{50} = 5\sqrt{2}$

b) $a = \langle -6,5 \rangle$

$\sqrt{(-6)^2 + (5)^2}$
$\sqrt{61}$

c) $m = \langle -3,11 \rangle$

$\sqrt{(-3)^2 + (11)^2}$
$\sqrt{130}$

3) Find the angle θ between vectors **u** and **v** to the nearest tenth of a degree.

a) $u = \langle -5,-2 \rangle$ and $v = \langle 4,4 \rangle$

$\cos\theta = \dfrac{-5 \cdot 4 + -2 \cdot 4}{\sqrt{(-5)^2 + (-2)^2} \bullet \sqrt{(4)^2 + (4)^2}}$

$\cos\theta = \dfrac{-28}{\sqrt{29}\,\sqrt{32}}$ $\theta = 156.80^\circ$

b) $u = \langle 9,5 \rangle$ and $v = \langle -6,7 \rangle$

$\cos\theta \quad \dfrac{9 \cdot -6 + 5 \cdot 7}{\sqrt{9^2 + 5^2} \bullet \sqrt{(-6)^2 + 7^2}}$

$\cos\theta = \dfrac{-19}{\sqrt{106} \cdot \sqrt{85}}$ $\theta = 101.55^\circ$

c) $u = \langle -3,-5 \rangle$ and $v = \langle 2,-3 \rangle$

$\cos\theta = \dfrac{-3 \cdot 2 + -5 \cdot -3}{\sqrt{(-3)^2 + (-5)^2} \bullet \sqrt{2^2 + (-3)^2}}$

$\cos\theta = \dfrac{9}{\sqrt{34} \cdot \sqrt{13}}$

$\theta = 64.65^\circ$

d) $u = \langle 1,-4 \rangle$ and $v = \langle 2,6 \rangle$

$\cos\theta = \dfrac{1 \cdot 2 + -4 \cdot 6}{\sqrt{1^2 + (-4)^2} \cdot \sqrt{2^2 + 6^2}}$

$\cos\theta = \dfrac{-22}{\sqrt{17} \cdot \sqrt{40}}$

$\theta = 147.53$

(1) $3 \cdot 6 + -5 \cdot 2$

$18 + -10$

8 no

(3) $9 \cdot 1 + -3 \cdot 3$

$9 + -9$

0 yes

(5) $1 \cdot 2 + -4 \cdot 8$

$2 + -32$

-30 no

(7) $-4 \cdot -5 + 6 \cdot -2$

$20 + -12$

8 no

(9) a) $406 \cdot 27.5 + 297 \cdot 15$

15620

b) total cost for both types of basketballs

(17) $\cos\theta = \dfrac{0 \cdot 1 + -5 \cdot -4}{\sqrt{0^2 + (-5)^2} \cdot \sqrt{1^2 + (-4)^2}}$

$\cos\theta = \dfrac{20}{5 \cdot \sqrt{17}}$

$\theta = 14.04°$

(19) component form $\langle -2, 3 \rangle$ & $\langle -4, -2 \rangle$

$\cos\theta = \dfrac{-2 \cdot -4 + 3 \cdot -2}{\sqrt{(-2)^2 + 3^2} \cdot \sqrt{(-4)^2 + (-2)^2}}$

$\cos\theta = \dfrac{2}{\sqrt{13} \cdot \sqrt{20}}$

$\theta = 82.87°$

(21) component form $\langle -1, -3 \rangle$ & $\langle -7, -3 \rangle$

$\cos\theta = \dfrac{-1 \cdot -7 + -3 \cdot -3}{\sqrt{(-1)^2 + (-3)^2} \cdot \sqrt{(-7)^2 + (-3)^2}}$

$\cos\theta = \dfrac{16}{\sqrt{10} \cdot \sqrt{58}}$

$\theta = 48.37°$

(23) component form $\langle -10, 1 \rangle$ & $\langle 10, -5 \rangle$

$\cos\theta = \dfrac{-10 \cdot 10 + 1 \cdot -5}{\sqrt{(-10)^2 + 1^2} \cdot \sqrt{10^2 + (-5)^2}}$

$\cos\theta = \dfrac{-105}{\sqrt{101} \cdot \sqrt{125}}$

$\theta = 159.15°$

Kuta Software - Infinite Precalculus

Name Key

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Two-Dimensional Vector Dot Products

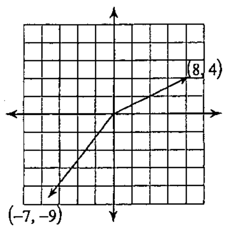
Find the dot product of the given vectors.

1) $\vec{u} = \langle 3, 9\rangle$
$\vec{v} = \langle 6, 5\rangle$

$3\cdot 6 + 9\cdot 5$
$18 + 45$
63

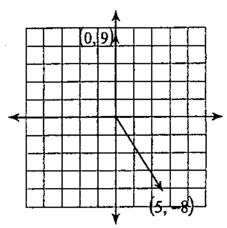
2) $\vec{u} = -\vec{i} + 5\vec{j}$
$\vec{v} = -6\vec{i} - 2\vec{j}$

3)



$-7\cdot 8 + -9\cdot 4$
-92

4)



angle between is 0° or 180°
dot product = 0

State if the two vectors are parallel, orthogonal, or neither.

5) $\vec{u} = \langle 4, -9\rangle$
$\vec{v} = \langle -9, 4\rangle$

$4\cdot -9 + -9\cdot 4$
$-36 + -36$
-72

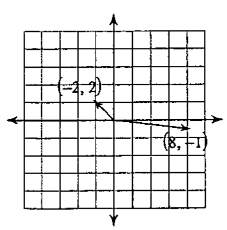
$\cos\theta = \frac{-72}{\sqrt{97}\cdot\sqrt{97}}$ $\theta = 137.92°$

neither

6) $\vec{u} = -5\vec{i} - 2\vec{j}$
$\vec{v} = -10\vec{i} + 25\vec{j}$

Find the measure of the angle between the two vectors.

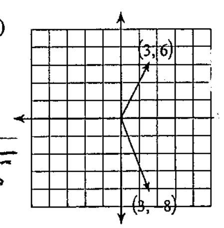
7)



$\cos\theta = \frac{-2\cdot 8 + 2\cdot 1}{\sqrt{8}\cdot\sqrt{65}}$

$\theta = 142.13°$

8)



5

9) $\vec{u} = \langle -8, -2\rangle$
$\vec{v} = \langle -3, 3\rangle$

$\cos\theta = \frac{-8\cdot -3 + -2\cdot 3}{\sqrt{(-8)^2 + (-2)^2}\cdot\sqrt{(-3)^2 + 3^2}}$

$\cos\theta = \frac{18}{\sqrt{68}\cdot\sqrt{18}}$ $\theta = 59.04°$

10) $\vec{u} = -8\vec{j}$
$\vec{v} = -9\vec{i} - 2\vec{j}$

Find the projection of u onto v.

11) $\vec{u} = \langle 8, 2\rangle$
$\vec{v} = \langle -7, -3\rangle$

$\frac{8\cdot -7 + 2\cdot -3}{\left(\sqrt{(-7)^2 + (-3)^2}\right)^2}\cdot\langle -7, -3\rangle$

$\frac{-62}{58}\langle -7, -3\rangle$

$\left\langle \frac{217}{29}, \frac{93}{29}\right\rangle$

12) $\vec{u} = 5\vec{i} - 5\vec{j}$
$\vec{v} = 7\vec{i} - 5\vec{j}$

Find the projection of u onto v. Then write u as the sum of two orthogonal vectors.

13) $\vec{u} = \langle -2, -3\rangle$
$\vec{v} = \langle -7, 9\rangle$

$\frac{-2\cdot -7 + -3\cdot 9}{\left(\sqrt{(-7)^2 + (9)^2}\right)^2}\cdot\langle -7, 9\rangle$

$\frac{-13}{130}\cdot\langle -7, 9\rangle$

$\langle .7, -.9\rangle$

$\vec{u} = \left\langle \frac{7}{10}, -\frac{9}{10}\right\rangle + \left\langle -\frac{27}{10}, \frac{-21}{10}\right\rangle$