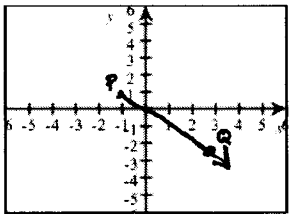
Pre-Calculus Name: Key

# *Vector Practice*

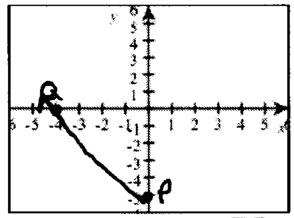
**1)** Find and draw the vector **v** with initial point $P$ and terminal point $Q$. Also find the magnitude of **v**.

**a)** $P(-1,1), Q(3,-2)$



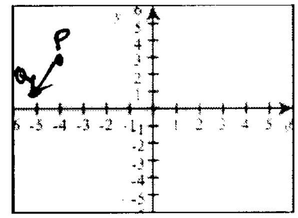
v = $\langle 4,-3\rangle$ |v| = $5$

**b)** $P(0,-5), Q(-4,0)$



v = $\langle -4,5\rangle$ |v| = $\sqrt{41}$

**c)** $P(-4,3), Q(-5,1)$



v = $\langle -1,-2\rangle$ |v| = $\sqrt{5}$

**2)** Show work to determine if the vector **v** with initial point $(p_1, p_2)$ and terminal point $(q_1,q_2)$ is equivalent to vector **w** with initial point $(r_1,r_2)$ and terminal point $(s_1,s_2)$

**a)** **v** $(5,3),(-2,2)$ **w**$(7,-1),(0,-2)$

$\vec{v} = \langle -2-5, 2-3\rangle = \langle -7,-1\rangle$
$\vec{w} = \langle 0-7, -2-^-1\rangle = \langle -7,-1\rangle$ equal

**b)** **v** $(-10,-3),(-1,-12)$ **w**$(7,-1),(-2,8)$

$\vec{v} = \langle -1-^-10, -12-^-3\rangle = \langle 9,-9\rangle$
$\vec{w} = \langle -2-7, 8-^-1\rangle = \langle -9,9\rangle$ not equal

**3)** Given the vectors $\mathbf{u} = \langle -1,7\rangle$, $\mathbf{v} = \langle 3,-1\rangle$, find the following:

**a)** $\mathbf{u}+\mathbf{v}$

$\langle 2,6\rangle$

**b)** $\mathbf{u}-\mathbf{v}$

$\langle -4,8\rangle$

**c)** $4\mathbf{u}-3\mathbf{v}$

$\langle -13,31\rangle$

**d)** $\mathbf{u}\cdot\mathbf{v}$

dot product = $-10$

**e)** $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\frac{-10}{(\sqrt{10})^2} = \frac{-10}{10} = -1$

$\langle -3,1\rangle$

**f)** write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$)

$\langle -3,1\rangle + \langle 2,6\rangle$

**g)** the angle between **u** and **v**

$\theta = 116.57^\circ$

**4)** Find a unit vector in the direction of the following vectors and show that it has length 1.

**a)** $\mathbf{v} = \langle 8,-15\rangle$

$\langle \frac{8}{17}, \frac{-15}{17}\rangle$

**b)** $\mathbf{v} = \langle 3,0\rangle$

$\langle 1,0\rangle$

**c)** $\mathbf{v} = \langle -4\sqrt{2},-2\rangle$

$\langle -\frac{2\sqrt{2}}{3}, \frac{-1}{3}\rangle$

**5)** Let **u** be the vector with initial point $(1,-8)$ and terminal point $(-1,-5)$ and let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. Write the following as a linear combination of **i** and **j**.

**a)** $-2\mathbf{u}$

$4i - 6j$

**b)** $\mathbf{u} - 2\mathbf{v}$

$-8i + 11j$

**c)** $\frac{\mathbf{u}}{|\mathbf{v}|}$

$-\frac{2}{5}i + \frac{3}{5}j$

**6)** Write the vector **v** given its magnitude and direction angle.

**a)** $|\mathbf{v}| = 6 \quad \theta = 45°$

$\langle 3\sqrt{2}, 3\sqrt{2} \rangle$

**b)** $|\mathbf{v}| = 12 \quad \theta = 240°$

$\langle -6, -6\sqrt{3} \rangle$

**c)** $|\mathbf{v}| = 10 \quad \theta =$ direction of $6\mathbf{i} - 2\mathbf{j}$

$10 \cdot \left\langle \frac{6}{2\sqrt{10}}, \frac{-2}{2\sqrt{10}} \right\rangle$

$10 \cdot \left\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle$

$\langle 3\sqrt{10}, -\sqrt{10} \rangle$

**7)** A plane is flying on a bearing of 295° at 360 mph. A wind is blowing with the bearing 320° at 38mph.

**a)** Write a vector (in component form) of the velocity produced by the airplane alone.

p = $\langle -326.27, 152.14 \rangle$

**b)** Write a vector (in component form) of the velocity of the wind.

w = $\langle -24.43, 29.11 \rangle$

**c)** Write a vector (in component form) of the actual velocity of the plane.

v = $\langle -350.70, 181.25 \rangle$

**d)** Find the actual speed and direction angle (not the bearing) of the plane.

speed = 394.77 mph $\quad \theta =$ 152.67°

**8)** A boat is traveling at a bearing of 95° at 25 knots for 2 hours, then it changes direction to a bearing of 135° for 3 hours. What is their bearing and distance from the original starting place?

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**9)** Find the vector projection **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle -5, -2 \rangle$ & $\mathbf{v} = \langle -11, 3 \rangle$

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{539}{130}, \frac{147}{130} \right\rangle$

$\mathbf{u} = \left\langle -\frac{539}{130}, \frac{147}{130} \right\rangle + \left\langle \frac{-111}{130}, \frac{-407}{130} \right\rangle$

Pre-Calculus Name: Key

# *Vector Practice*

1) Find and draw the vector **v** with initial point $P$ and terminal point $Q$. Also find the magnitude of **v.**

a) $P(-1,1), Q(3,-2)$

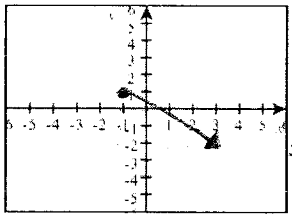
$\vec{V} = \langle 3-^-1, -2-1 \rangle$

$\vec{V} = \langle 4, -3 \rangle$

$|\vec{V}|$

$= \sqrt{16+9}$

$= \sqrt{25} = 5$



$\mathbf{v} = \langle 4, -3 \rangle$ $|\mathbf{v}| = 5$

b) $P(0,-5), Q(-4,0)$

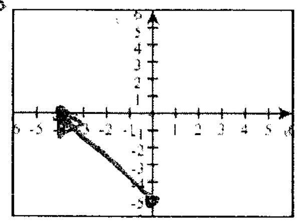
$\vec{V} = \langle -4-0, 0-^-5 \rangle$

$= \langle -4, 5 \rangle$

$|\vec{V}| =$

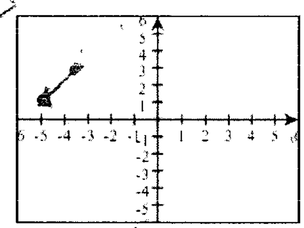
$= \sqrt{16+25}$

$= \sqrt{41}$



$\mathbf{v} = \langle -4, 5 \rangle$ $|\mathbf{v}| = \sqrt{41}$

c) $P(-4,3), Q(-5,1)$



$\mathbf{v} = \langle -1, -2 \rangle$ $|\mathbf{v}| = \sqrt{5}$

2) Show work to determine if the vector **v** with initial point $(p_1, p_2)$ and terminal point $(q_1, q_2)$ is equivalent to vector **w** with initial point $(r_1, r_2)$ and terminal point $(s_1, s_2)$

a) **v** $(5,3),(-2,2)$ **w**$(7,-1),(0,-2)$

$\vec{V} = \langle -2-5, 2-3 \rangle$ $\vec{W} = \langle 0-7, -2-^-1 \rangle$

$= \langle -7, -1 \rangle$ $= \langle -7, -1 \rangle$ Equivalent

b) **v** $(-10,-3),(-1,-12)$ **w**$(7,-1),(-2,8)$

$\vec{V} = \langle -1-^-10, -12-^-3 \rangle$ $\vec{W} = \langle -2-7, 8-^-1 \rangle$

$= \langle 9, -9 \rangle$ $= \langle -9, 9 \rangle$ NOT EQUIVALENT

3) Given the vectors $\mathbf{u} = \langle -1, 7 \rangle$, $\mathbf{v} = \langle 3, -1 \rangle$, find the following:

a) $\mathbf{u} + \mathbf{v}$

$\langle -1, 7 \rangle + \langle 3, -1 \rangle$

$= \boxed{\langle 2, 6 \rangle}$

b) $\mathbf{u} - \mathbf{v}$

$\langle -1, 7 \rangle - \langle 3, -1 \rangle$

$\boxed{\langle -4, 8 \rangle}$

c) $4\mathbf{u} - 3\mathbf{v}$

$4\langle -1, 7 \rangle - 3\langle 3, -1 \rangle$

$\langle -4, 28 \rangle + \langle -9, 3 \rangle$

$\boxed{\langle -13, 31 \rangle}$

d) $\mathbf{u} \cdot \mathbf{v}$

$\langle -1, 7 \rangle \cdot \langle 3, -1 \rangle$

$-3 + -7 = \boxed{-10}$

e) $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\dfrac{\langle -1, 7 \rangle \cdot \langle 3, -1 \rangle}{\langle 3, -1 \rangle \cdot \langle 3, -1 \rangle} \langle 3, -1 \rangle$

$= \dfrac{-3 + -7}{9+1} \vec{v} = \dfrac{-10}{10} \vec{v} = -1\langle 3, -1 \rangle$

$= \boxed{\langle -3, 1 \rangle}$

f) write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$)

$\boxed{\vec{U} = \langle -3, 1 \rangle + \langle 2, 6 \rangle}$

$\langle -1, 7 \rangle - \langle -3, 1 \rangle$

$= \langle 2, 6 \rangle$

g) the angle between **u** and **v**

$\cos^{-1}\left(\dfrac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|}\right) = \cos^{-1}\left(\dfrac{-10}{\sqrt{50} \cdot \sqrt{10}}\right)$

$\cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right) \approx \boxed{116.565°}$

4) Find a unit vector in the direction of the following vectors and show that it has length 1.

a) $\mathbf{v} = \langle 8, -15 \rangle$ $|\vec{V}| = \sqrt{289} = 17$

$\boxed{\vec{U} = \left\langle \dfrac{8}{17}, \dfrac{-15}{17} \right\rangle}$

$|\vec{U}| = \sqrt{\dfrac{64}{289} + \dfrac{225}{289}} = \sqrt{\dfrac{289}{289}} = 1$ ✓

b) $\mathbf{v} = \langle 3, 0 \rangle$ $|\vec{V}| = 3$

$\boxed{\vec{U} = \langle 1, 0 \rangle}$

$|\vec{U}| = \sqrt{1+0} = \sqrt{1} = 1$ ✓

c) $\mathbf{v} = \langle -4\sqrt{2}, -2 \rangle$ $|\vec{V}| = \sqrt{32+4} = \sqrt{36} = 6$

$\boxed{\vec{U} = \left\langle \dfrac{-2\sqrt{2}}{3}, \dfrac{-1}{3} \right\rangle}$

$|\vec{U}| = \sqrt{\dfrac{8}{9} + \dfrac{1}{9}} = \sqrt{\dfrac{9}{9}} = \sqrt{1} = 1$ ✓

5) Let **u** be the vector with initial point $(1, -8)$ and terminal point $(-1, -5)$ and let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. Write the following as a ~~linear combination of **i** and **j**.~~

$\vec{U} = \langle -2, 3 \rangle$ $\vec{V} = \langle 3, -4 \rangle$

$|\vec{v}| = \sqrt{9+16} = \sqrt{25} = 5$

a) $-2\mathbf{u}$

$= \boxed{4\vec{i} - 6\vec{j}}$

b) $\mathbf{u} - 2\mathbf{v}$

$\boxed{-8\vec{i} + 11\vec{j}}$

c) $\dfrac{\mathbf{u}}{|\mathbf{v}|} =$

$\left\langle -\dfrac{2}{5}, \dfrac{3}{5} \right\rangle$

**6)** Write the vector **v** given its magnitude and direction angle.

$\sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

**a)** $|\mathbf{v}| = 6 \quad \theta = 45°$

$= \langle 6\cos 45°, 6\sin 45° \rangle$

$= \langle 6\left(\frac{\sqrt{2}}{2}\right), 6\left(\frac{\sqrt{2}}{2}\right) \rangle$

$= \boxed{\langle 3\sqrt{2}, 3\sqrt{2} \rangle}$

**b)** $|\mathbf{v}| = 12 \quad \theta = 240°$

$\langle 12\cos 240°, 12\sin 240° \rangle$

$\langle 12\left(-\frac{1}{2}\right), 12\left(-\frac{\sqrt{3}}{2}\right) \rangle$

$= \boxed{\langle -6, -6\sqrt{3} \rangle}$

**c)** $|\mathbf{v}| = 10 \quad \theta =$ direction of $6\mathbf{i} - 2\mathbf{j}$

$10\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \rangle$

$= \boxed{\langle 3\sqrt{10}, -\sqrt{10} \rangle}$

**7)** A plane is flying on a bearing of 295° at 360 mph. A wind is blowing with the bearing 320° at 38mph.

**a)** Write a vector (in component form) of the velocity produced by the airplane alone.

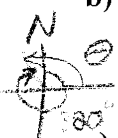
$\theta = 155°$

NOT 295°

**p** $= \langle 360\cos 155°, 360\sin 155° \rangle$

$\approx \langle -326.2708, 152.1426 \rangle$

**b)** Write a vector (in component form) of the velocity of the wind.



$\theta = 130°$ NOT 320°

**w** $= \langle 38\cos 130°, 38\sin 130° \rangle$

$\approx \langle -24.426, 29.110 \rangle$

**c)** Write a vector (in component form) of the actual velocity of the plane.

**v** $= \langle -350.697, 181.252 \rangle$

**d)** Find the actual speed and direction angle (not the bearing) of the plane.

$|\vec{v}| = \sqrt{(-350.697)^2 + 181.252^2} \approx 394.766$ speed = 394.8 θ = 152.7°

$\theta = \tan^{-1}\left(\frac{181.252}{-350.697}\right) = -27.33°$

$+180°$

(need a 2nd quadrant angle)

**8)** Find the vector projection **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle -5, -2 \rangle$ & $\mathbf{v} = \langle -11, 3 \rangle$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$

$\frac{\langle -5, -2 \rangle \cdot \langle -11, 3 \rangle}{\langle -11, 3 \rangle \cdot \langle -11, 3 \rangle}\vec{v} = \frac{55 + -6}{121 + 9} = \frac{49}{130}\langle -11, 3 \rangle$

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle \frac{-539}{130}, \frac{147}{130} \rangle$

$\mathbf{u} = \langle \frac{-539}{130}, \frac{147}{130} \rangle + \langle \frac{-111}{130}, \frac{-407}{130} \rangle$