Practice 6.3 Volume: The Washer Method

1) Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

a) $y = 4 - x^2, x = 0, y = 0$
(use the 1st quadrant portion)

$$\pi \int_0^2 (4-x^2)^2 dx = \frac{256}{15}\pi$$

b) $y = x^2, y = x^3$

$$\pi \int_0^1 \left[(x^2)^2 - (x^3)^2\right] dx = \frac{2}{35}\pi$$

2) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the following lines.

a) the y-axis

$$\pi \int_0^2 \left[(4)^2 - (y^2)^2\right] dy = \frac{128}{5}\pi$$

b) the line $x = 6$

$$\pi \int_0^2 \left[(6-y^2)^2 - (2)^2\right] dy = \frac{192}{5}\pi$$

3) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = x^2$ and $y = 4x - x^2$ about the following lines.

a) the x-axis

$$\pi \int_0^2 \left[(4x-x^2)^2 - (x^2)^2\right] dx = \frac{32}{3}\pi$$

b) the line $y = 6$

$$\pi \int_0^2 \left[(6-x^2)^2 - (6-(4x-x^2))^2\right] dx = \frac{64}{3}\pi$$

4) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 4$.

a) $y = x$, $y = 3$, $x = 0$

$$\pi \int_0^3 \left[(4-x)^2 - (1)^2\right] dx = 18\pi$$

b) $y = \frac{1}{1+x}$, $y = 0$, $x = 0$, $x = 3$

$$\pi \int_0^3 \left[(4)^2 - \left(4 - \frac{1}{1+x}\right)^2\right] dx \approx 32.485$$

5) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $x = y^2$ and $x = 4$ about the line $x = 6$.

$$\pi \int_{-2}^2 \left[(6-y^2)^2 - (2)^2\right] dy = \frac{384}{5}\pi$$

6) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = x^2 + 1$, $y = -x^2 + 2x + 5$, $x = 0$ and $x = 3$ about the x-axis.

$$\pi \int_0^2 \left[(-x^2+2x+5)^2 - (x^2+1)^2\right] dx + \pi \int_2^3 \left[(x^2+1)^2 - (-x^2+2x+5)^2\right] dx$$

$$= \frac{277}{3}\pi$$