

Practice--Weighted Voting (Banzhaf and Shapley-Shubik)

1. Find the Banzhaf Power index for the weighted voting system: [101: 99, 98, 3]

winning coalition	wt	critical voter
P_1, P_2	197	P_1, P_2
P_1, P_3	102	P_1, P_3
P_2, P_3	101	P_2, P_3
P_1, P_2, P_3	200	none

Banzhaf Coalitions: 3 Players

$\{P_1\} 99$ $\{P_1, P_2\} 197$ $\{P_1, P_2, P_3\} 200$
 $\{P_2\} 98$ $\{P_1, P_3\} 102$
 $\{P_3\} 3$ $\{P_2, P_3\} 101$

$$P_1 = \frac{2}{6} = \frac{1}{3}$$

$$P_2 = \frac{2}{6} = \frac{1}{3}$$

$$P_3 = \frac{2}{6} = \frac{1}{3}$$

2. Find the Banzhaf Power Distribution for [10: 8, 4, 2, 1]

winning	wt	critical
P_1, P_2	12	P_1, P_2
P_1, P_3	10	P_1, P_3
P_1, P_2, P_3	14	P_1
P_1, P_2, P_4	13	P_1, P_2
P_1, P_3, P_4	11	P_1, P_3
P_1, P_2, P_3, P_4	15	P_1

Banzhaf Coalitions: 4 Players

$\{P_1\} 8$ $\{P_1, P_2\} 12$ $\{P_1, P_2, P_3\} 14$
 $\{P_2\} 4$ $\{P_1, P_3\} 10$ $\{P_1, P_2, P_4\} 13$
 $\{P_3\} 2$ $\{P_1, P_4\} 9$ $\{P_1, P_3, P_4\} 11$
 $\{P_4\} 1$ $\{P_2, P_3\} 6$ $\{P_2, P_3, P_4\} 7$
 $\{P_2, P_4\} 5$ $\{P_1, P_2, P_3, P_4\} 15$
 $\{P_3, P_4\} 3$

$$P_1 = \frac{6}{10} = 60\%$$

$$P_2 = \frac{2}{10} = 20\%$$

$$P_3 = \frac{2}{10} = 20\%$$

$$P_4 = 0\%$$

3. A weighted voting system has three players. The sequential coalitions are listed below with the pivotal player underlined in each. Find the Shapley-Shubik Power Index for the weighted voting system.

$$P_1 = \frac{3}{6} = \frac{1}{2}$$

$$P_2 = \frac{3}{6} = \frac{1}{2}$$

$$P_3 = \frac{0}{6} = 0$$

$$\langle P_1, \underline{P_2}, P_3 \rangle, \langle P_1, P_3, \underline{P_2} \rangle, \langle P_2, \underline{P_1}, P_3 \rangle$$

$$\langle P_2, P_3, \underline{P_1} \rangle, \langle P_3, P_1, \underline{P_2} \rangle, \langle P_3, P_2, \underline{P_1} \rangle$$

4. A weighted voting system has four players. The sequential coalitions are listed below with the pivotal player underlined in each. Find the Shapley-Shubik Power Distribution for the weighted voting system.

$$P_1 = \frac{14}{24} = 58\frac{1}{3}\%$$

$$P_2 = \frac{6}{24} = 25\%$$

$$P_3 = \frac{2}{24} = 8\frac{1}{3}\%$$

$$P_4 = \frac{2}{24} = 8\frac{1}{3}\%$$

$$\langle P_1, \underline{P_2}, P_3, P_4 \rangle, \langle P_2, \underline{P_1}, P_3, P_4 \rangle, \langle P_3, P_1, \underline{P_2}, P_4 \rangle, \langle P_4, P_1, P_2, \underline{P_3} \rangle$$

$$\langle P_1, \underline{P_2}, P_4, P_3 \rangle, \langle P_2, \underline{P_1}, P_4, P_3 \rangle, \langle P_3, P_1, \underline{P_4}, P_2 \rangle, \langle P_4, P_1, P_2, \underline{P_3} \rangle$$

$$\langle P_1, P_3, \underline{P_2}, P_4 \rangle, \langle P_2, P_3, \underline{P_1}, P_4 \rangle, \langle P_3, P_2, \underline{P_1}, P_4 \rangle, \langle P_4, P_2, P_1, \underline{P_3} \rangle$$

$$\langle P_1, P_3, \underline{P_4}, P_2 \rangle, \langle P_2, P_3, P_4, \underline{P_1} \rangle, \langle P_3, P_2, P_4, \underline{P_1} \rangle, \langle P_4, P_2, P_3, \underline{P_1} \rangle$$

$$\langle P_1, P_4, \underline{P_2}, P_3 \rangle, \langle P_2, P_4, \underline{P_1}, P_3 \rangle, \langle P_3, P_4, \underline{P_1}, P_2 \rangle, \langle P_4, P_3, P_1, \underline{P_2} \rangle$$

$$\langle P_1, P_4, \underline{P_3}, P_2 \rangle, \langle P_2, P_4, P_3, \underline{P_1} \rangle, \langle P_3, P_4, P_2, \underline{P_1} \rangle, \langle P_4, P_3, P_2, \underline{P_1} \rangle$$

5. Find the Shapley-Shubik Power Distribution for [4: 3, 2, 1]

$$P_1 = \frac{4}{6} = 66\frac{2}{3}\%$$

$$P_2 = \frac{1}{6} = 16\frac{2}{3}\%$$

$$P_3 = \frac{1}{6} = 16\frac{2}{3}\%$$

Sequential

Coalitions:

3 Players

[P₁(P₂)P₃]

[P₁(P₃)P₂]

[P₂(P₁)P₃]

[P₂,P₃(P₁)]

[P₃(P₁)P₂]

[P₃,P₂(P₁)]

6. Find the Shapley-Shubik Power Distribution for [10: 8, 4, 2, 1]

$$P_1 = \frac{16}{24} = 66\frac{2}{3}\%$$

$$P_2 = \frac{4}{24} = 16\frac{2}{3}\%$$

$$P_3 = \frac{4}{24} = 16\frac{2}{3}\%$$

$$P_4 = \frac{0}{24} = 0\%$$

Sequential Coalitions: 4 Players

[P₁(P₂)P₃,P₄] [P₂(P₁)P₃,P₄] [P₃(P₁)P₂,P₄] [P₄,P₁(P₂)P₃]

[P₁(P₂)P₄,P₃] [P₂(P₁)P₄,P₃] [P₃(P₁)P₄,P₂] [P₄,P₁(P₃)P₂]

[P₁(P₃)P₂,P₄] [P₂,P₃(P₁)P₄] [P₃,P₂(P₁)P₄] [P₄,P₂(P₁)P₃]

[P₁(P₃)P₄,P₂] [P₂,P₃,P₄(P₁)] [P₃,P₂,P₄(P₁)] [P₄,P₂,P₃(P₁)]

[P₁,P₄(P₂)P₃] [P₂,P₄(P₁)P₃] [P₃,P₄(P₁)P₂] [P₄,P₃(P₁)P₂]

[P₁,P₄(P₃)P₂] [P₂,P₄,P₃(P₁)] [P₃,P₄,P₂(P₁)] [P₄,P₃,P₂(P₁)]