* 2019 has different problem #'s

Pre- Calculus

Name: KEY Spring 2018

REVIEW FOR MIDTERM

Chapter 1 – Functions and their properties:

1) State the domain of each of the functions below:

a) $f(x) = \sqrt{3x-4}$

$3x-4 \geq 0$

$x \geq \frac{4}{3}$

$\left[\frac{4}{3}, \infty\right)$

b) $f(x) = \dfrac{\sqrt{7-x}}{x-4}$

$7-x \geq 0$

$-x \geq -7$ $\quad x-4 \neq 0$

$x \leq 7$ $\quad x \neq 4$

$(-\infty, 4) \cup (4, 7]$

c) $f(x) = \ln(3x-7)$

$3x-7 > 0$

$x > \frac{7}{3}$

$\left(\frac{7}{3}, \infty\right)$

d) $f(x) = \dfrac{\sqrt{3-x}}{2x^2+x-3}$

$3-x \geq 0$ $\quad (2x+3)(x-1)$

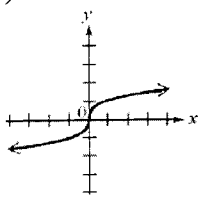
$-x \geq -3$ $\quad x \neq -\frac{3}{2}, 1$

$x \leq 3$

$\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, 1\right) \cup (1, 3]$

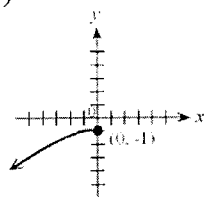
2) State the interval over which each of the following are increasing/decreasing:

a)



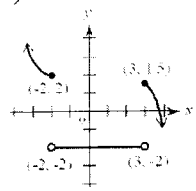
incr $(-\infty, \infty)$

b)



incr $(-\infty, 0]$

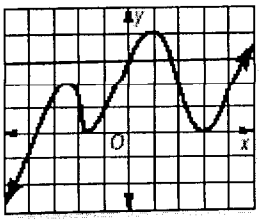
c)



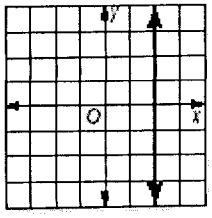
decr $(-\infty, -2) \cup (3, \infty)$

3) Which of the following are functions? A & D

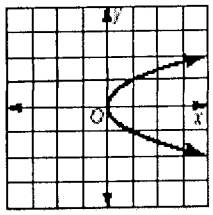
a)



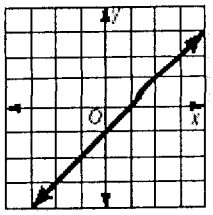
b)



c)



d)



4) Determine whether each of the following are even, odd, or neither:

a) $f(x) = \dfrac{3x}{x^2-4}$

$f(-x) = \dfrac{3(-x)}{(-x)^2-4} = \dfrac{-3x}{x^2-4}$

ODD

b) $y = 5|x| - 7x^4 + 11$

$5|-x| - 7(-x)^4 + 11$

$5|-x| - 7x^4 + 11$

$|-x|$ will give same values as $|x|$

EVEN

c) $f(x) = 5x^7 - 4x^3 - x$

$f(-x) = 5(-x)^7 - 4(-x)^3 - (-x)$

$= -5x^7 + 4x^3 + x$

ODD

d) $y = \sqrt{2x-9}$

$\sqrt{2(-x)-9}$

$\sqrt{-2x-9}$

NEITHER

5) Determine the functions implicitly defined below:

a) $x^2 + y^2 = 9$

$y^2 = -x^2 + 9$

$\boxed{y = \pm\sqrt{-x^2+9}}$

b) $y^2 - x = 2$

$y^2 = x + 2$

$\boxed{y = \pm\sqrt{x+2}}$

c) $9x^2 + 16y^2 = 100 - 24xy$

$9x^2 + 24xy + 16y^2 = 100$

$(3x+4y)(3x+4y) = 100$

$(3x+4y)^2 = 100$

$3x + 4y = \pm 10$

$4y = -3x \pm 10$

$\boxed{y = \frac{-3x \pm 10}{4}}$

6) Use the given functions to find the compositions and their domains

$f(x) = x^2 - 3x \qquad g(x) = \frac{3}{x+1} \qquad h(x) = 2\sqrt{1-x} \qquad k(x) = \sqrt{x+3}$

a) $(f \circ k)(x)$

$f(k(x))$

$(\sqrt{x+3})^2 - 3\sqrt{x+3}$ $\quad x+3 \geq 0$, $x \geq -3$

$\boxed{x+3-3\sqrt{x+3}}$

$\boxed{D: [-3, \infty)}$

b) $(h \circ g)(x)$

$h(g(x))$

$2\sqrt{1 - \frac{3}{x+1}}$

$2\sqrt{\frac{x+1-3}{x+1}}$

$2\sqrt{\frac{x-2}{x+1}}$

$\frac{x-2}{x+1} \geq 0$

$+ \quad 0 \quad - \quad \bullet \quad +$

$-1 \quad 2$

$(-\infty, -1) \cup [2, \infty)$

c) $g(f(x))$

$\frac{3}{x^2-3x+1}$

$x^2 - 3x + 1 \neq 0$

$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$\boxed{x \neq \frac{3 \pm \sqrt{5}}{2}}$

d) $f(h(x))$

$(2\sqrt{1-x})^2 - 3(2\sqrt{1-x})$

$4(1-x) - 6\sqrt{1-x}$

$\boxed{4 - 4x - 6\sqrt{1-x}}$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$ $\boxed{(-\infty, 1]}$

7) Each function below is a transformation of one of the 12 basic functions. Describe the transformations.

a) $f(x) = e^{-2x+8} - 7$

$= e^{-2(x-4)} - 7$

right 4

horiz. shrink $* \frac{1}{2}$

refl. over y-axis

down 7

b) $g(x) = \frac{1}{2 + 2e^{-x+5}} = \frac{1}{2}\left(\frac{1}{1+e^{-(x-5)}}\right)$

Shift right 5

vert. shrink $* \frac{1}{2}$

c) $f(x) = 9\cos(2x) - 11$

horiz. shrink $* \frac{1}{2}$

vert. stretch $* 9$

down 11

8) Given the parametric functions below, eliminate the parameter:

<skip>

a) $y = 3t^2 - 11$

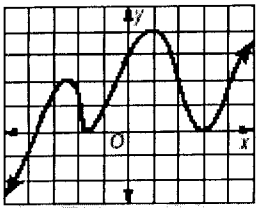
$x = -2t + 7$

b) $y = 3|t| + 20$

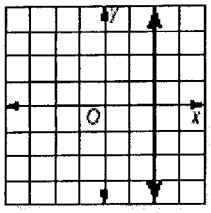
$x = 4t - 3$

9) Which of the following have inverses that are functions? B, C & D

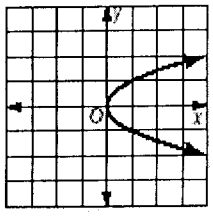
a)



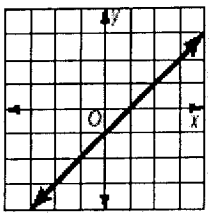
b)



c)



d)



Chapter 2A – Polynomial & Power Functions:

1) Write the following quadratic functions in vertex form then state the vertex:

a) $f(x) = x^2 - 8x + 11$

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$= x^2 - 8x + 16 + 11 - 16$

$f(x) = (x-4)^2 - 5$

$V(4, -5)$

b) $y = 3x^2 + 6x + 5$

$\left(\frac{2}{2}\right)^2 = (1)^2 = 1$

$= 3(x^2 + 2x + 1) + 5 - 3$

$y = 3(x+1)^2 + 2$

$V(-1, 2)$

2) Find the zeros of the polynomial functions below:

a) $f(x) = x^3 - 3x^2 - 25x + 75$

$x^2(x-3) - 25(x-3) = 0$

$(x^2 - 25)(x-3) = 0$

$(x+5)(x-5)(x-3) = 0$

$x = 5, -5, 3$

b) $f(x) = x^4 - 11x^2 + 28$

$0 = (x^2 - 7)(x^2 - 4)$

$x^2 - 7 = 0 \quad x^2 - 4 = 0$

$x = \pm\sqrt{7} \quad x = \pm 2$

c) $f(x) = x^3 - 2x^2 - 5x + 6$

possible roots: $\pm 1, \pm 2, \pm 3, \pm 6$

	1	-2	-5	6
1		1	-1	-6
	1	-1	-6	0

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 1, 3, -2$

3) Use long division or synthetic (you should look in your notes to determine which to use when) to find:

a) $\frac{3x^2 - 7x + 2}{x+5}$

-5	3	-7	2
		-15	110
	3	-22	112

$3x - 22 + \frac{112}{x+5}$

b) $\frac{2x^4 - x^3 - 2}{2x^2 + x + 1}$

$$\begin{array}{r} x^2 - x \\ 2x^2 + x + 1 \overline{\big) 2x^4 - x^3 + 0x^2 + 0x - 2} \\ -(2x^4 + x^3 + x^2) \\ \hline -2x^3 - x^2 + 0x \\ -(-2x^3 - x^2 - x) \\ \hline x - 2 \end{array}$$

$x^2 - x + \frac{x-2}{2x^2 + x + 1}$

4) State the complex conjugate of each of the following and then find their product:

a) $7 - 3i$ $\quad 7 + 3i$

$(7 - 3i)(7 + 3i)$

$49 + 21i - 21i - 9i^2$

58

b) $-5 + 8i$ $\quad -5 - 8i$

$(-5 + 8i)(-5 - 8i)$

$25 + 40i - 40i - 64i^2$

89

5) Find all the complex zeros of the following polynomial functions:

a) $y = x^3 - 2x^2 + 4x - 8$

$x^2(x-2) + 4(x-2) = 0$

$(x-2)(x^2+4) = 0$

$x - 2 = 0 \quad x^2 + 4 = 0$

$x = 2 \quad x = \pm\sqrt{-4} = \pm 2i$

b) $f(x) = x^2 - 6x + 25$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$

$= \frac{6 \pm \sqrt{36 - 100}}{2}$

$= \frac{6 \pm \sqrt{-64}}{2}$

$= \frac{6 \pm 8i}{2} = 3 \pm 4i$

c) $y = 7x^2 + 42$

$7x^2 + 42 = 0$

$7x^2 = -42$

$x^2 = -6$

$x = \pm\sqrt{-6} = \pm i\sqrt{6}$

6) Solve the following non-linear inequalities:

a) $(3x-4)^2(x+5)(x-3) > 0$ pos

$+ \quad - \quad - \quad +$

$-5 \quad 4/3 \quad 3$

$+-- \quad ++- \quad ++- \quad +++$

$\boxed{(-\infty,-5) \cup (3,\infty)}$

b) $x^3+9x^2+11x \le 21$

$x^3+9x^2+11(x-21) \le 0$

possible roots: $\pm1, \pm3, \pm7, \pm21$

$\begin{array}{r|rrrr} 1 & 1 & 9 & 11 & -21 \\ & & 1 & 10 & 21 \\ \hline & 1 & 10 & 21 & 0 \end{array}$

$x^2+10x+21=0$

$(x+7)(x+3)=0$

$x=-7,-3$

$(x-1)(x+7)(x+3) \le 0$

$- \quad + \quad - \quad +$

$-7 \quad -3 \quad 1$

$--- \quad -++ \quad --+ \quad +++$

$\boxed{(-\infty,-7] \cup [-3,1]}$

c) $\dfrac{x-8}{x^2-4} \ge \dfrac{-1}{x+2}$

$\dfrac{x-8}{(x+2)(x-2)} + \dfrac{1}{x+2} \ge 0$

$\dfrac{x-8+1(x-2)}{(x+2)(x-2)} \ge 0$

$\dfrac{2x-10}{(x+2)(x-2)} \ge 0$

$\dfrac{2(x-5)}{(x+2)(x-2)} \ge 0$

zero: 5 undef: -2,2

$- \quad + \quad - \quad +$

$-2 \quad 2 \quad 5$

$\boxed{(-2,2) \cup [5,\infty)}$

7) Determine whether each of the following is a polynomial. If it is state the degree & leading coefficient (if not put "n/a"):

a. $f(x)=5x^{3.2}+4x-3$

This is not a polynomial. (is/is not)

~~Degree = ____ & Leading coeff. = ____~~

~~Leading coeff. = ____~~

b. $f(x)=7x^2-3x+8$

This is a polynomial. (is/is not)

Degree = 2 & Leading coeff. = 7

c. $f(x)=\dfrac{7x+5}{3x}$

This is not a polynomial. (is/is not)

~~Degree = ____ & Leading coeff. = ____~~

d. $f(x)=\dfrac{7x+5}{3} = \dfrac{7}{3}x+\dfrac{5}{3}$

This is a polynomial. (is/is not)

Degree = 1 & Leading coeff. = $\dfrac{7}{3}$

e. $f(x)=\sqrt[3]{18x^3+4x^2+10}$

This is not a polynomial. (is/is not)

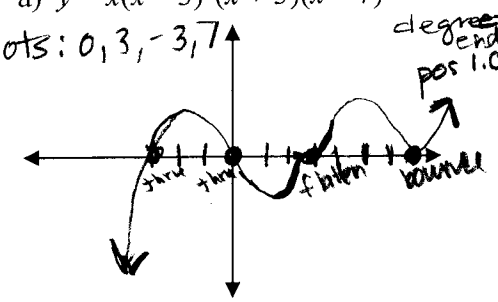
~~Degree = ____ & Leading coeff. = ____~~

8) Sketch each of the following polynomials:

a) $y=x(x-3)^3(x+3)(x-7)^2$

roots: 0, 3, -3, 7

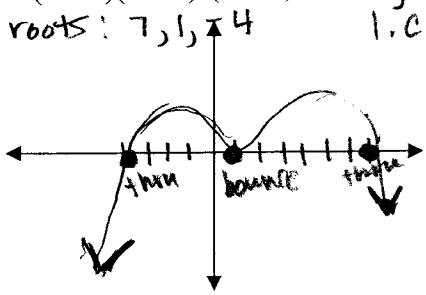
degree = 7 ends opposite pos l.c.



b) $f(x)=(7-x)(x-1)^2(x+4)$

roots: 7, 1, -4

degree = 4 ends same l.c. = neg



Chapter 2B – Rational Functions & Non-Linear Inequalities:

1) Simplify each of the following rational expressions:

a) $\dfrac{x^2+6x+5}{x^2+3x-10}$

$\dfrac{(x+1)(x+5)}{(x+5)(x-2)} = \boxed{\dfrac{x+1}{x-2}}$

$\boxed{x \ne -5, 2}$

b) $\dfrac{20x^3+30x^2+45x}{40x^4-135x}$

$\dfrac{5x(4x^2+6x+9)}{5x(8x^3-27)}$

$\dfrac{4x^2+6x+9}{(2x-3)(4x^2+6x+9)} = \boxed{\dfrac{1}{2x-3}}$

$\boxed{x \ne 0, 3/2}$

c) $\dfrac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}} = \dfrac{\frac{5}{x-2}}{\frac{1(x+1)+2(x-2)}{(x-2)(x+1)}}$

LCD = $(x-2)(x+1)$

$= \dfrac{5}{x-2} \cdot \dfrac{(x-2)(x+1)}{3x-3}$

$= \dfrac{5}{x-2} \cdot \dfrac{(x-2)(x+1)}{3(x-1)}$

$\boxed{\dfrac{5(x+1)}{3(x-1)}}$ $\boxed{x \ne 2, -1, 1}$

2) Determine each of the x- & y-intercepts and state the left and right end behavior for each of the following:

a. $f(x) = x^4 - 5x^2 + 4$

$x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$(x-2)(x+2)(x-1)(x+1) = 0$

y-int: $(0,4)$

Zero(s): $(2,0)(-2,0)(1,0)(-1,0)$

LEB: ∞

REB: ∞

b. $f(x) = x^4 - 2x^3 + 8x - 16$

$x^3(x-2) + 8(x-2) = 0$

$(x-2)(x^3+8) = 0$

$(x-2)(x+2)(x^2-2x+4)$

y-int: $(0,-16)$

Zero(s): $(2,0)(-2,0)$

LEB: ∞

REB: ∞

c. $f(x) = 81x^4 - 16$

$(9x^2+4)(9x^2-4) = 0$

$(9x^2+4)(3x+2)(3x-2) = 0$

$x = -2/3, 2/3$

y-int: $(0,-16)$

Zero(s): $(-2/3,0)\ (2/3,0)$

LEB: ∞

REB: ∞

3) Write the equations of all of the asymptotes of the function:

a) $y = \dfrac{5x^3 - 4x^2 + 1}{x^3 - 8}$

$(x-2)(x^2+2x+4)$

V.A. $x = 2$

H.A. $y = 5$

b) $y = \dfrac{5x^2 - 20x^2 + 20}{x^3 - 8} = \dfrac{-15x^2+20}{x^3-8}$

$= \dfrac{-5(3x^2-4)}{(x-2)(x^2+2x+4)}$

V.A. $x = 2$

H.A. $y = 0$

c) $y = \dfrac{x-10}{x^2-9} = \dfrac{x-10}{(x+3)(x-3)}$

V.A. $x = -3, x = 3$

H.A. $y = 0$

4) Solve each of the following rational equations

a) $\dfrac{x-2}{x+4} + \dfrac{x+1}{x+6} = \dfrac{11x+32}{x^2+10x+24}$ $(x+6)(x+4)$

LCD $= (x+4)(x+6)$

$(x-2)(x+6) + (x+1)(x+4) = 11x+32$

$x^2 + 4x - 12 + x^2 + 5x + 4 = 11x + 32$

$2x^2 - 2x - 40 = 0$

$2(x^2 - x - 20) = 0$

$2(x-5)(x+4) = 0$

$\boxed{x = 5}, \cancel{-4}$

b) $\dfrac{y+3}{y+2} = 1 - \dfrac{y+1}{y+2}$

LCD $= y+2$

$y+3 = y+2-(y+1)$

$y+3 = y+2-y-1$

$y+3 = 1$

$y = \cancel{-2}$ no soln

Chapter 3 – Logistic, Exponential, & Logarithmic Functions:

1) Write each of the following in exponential form:

a) $\log 7 = x$

$10^x = 7$

b) $\log_4 16 = 2$

$4^2 = 16$

c) $\log_u k = z$

$u^z = k$

2) Write each of the following in logarithmic form:

a) $7^3 = 343$

$\log_7 343 = 3$

b) $11^x = 3$

$\log_{11} 3 = x$

c) $t^4 = 11$

$\log_t 11 = 4$

3) Evaluate each of the following logarithms:

a) $\log_8 4 =$ $2/3$

$8^x = 4$

$2^{3x} = 2^2$

$3x = 2$ $x = 2/3$

b) $\log_9 27^{1/3} =$ $1/2$

$9^x = 27^{1/3}$

$3^{2x} = 3$

$2x = 1$ $x = 1/2$

c) $\ln e^{4x} =$ $4x$

d) $\log 0.001 =$ -3

$10^x = .001$

$10^x = \dfrac{1}{10^3} = 10^{-3}$

$x = -3$

4) Expand each of the following logarithms:

a) $\ln(7xy)$

$\ln 7 + \ln x + \ln y$

b) $\ln\left(\frac{x(x+4)}{\sqrt{x^2+1}}\right)$

$\ln x + \ln(x+4) - \ln(x^2+1)^{1/2}$

$\ln x + \ln(x+4) - \frac{1}{2}\ln(x^2+1)$

c) $\log\left(\frac{x^3y^6}{\sqrt{z}}\right)$

$3\log x + 6\log y - \frac{1}{2}\log z$

5) Condense each of the following logarithms:

a) $\log_2 x - 2\log_2 y + \frac{1}{3}\log_2 t$

$\log_2 x - \log_2 y^2 + \log_2 t^{1/3}$

$\log_2\left(\frac{x \cdot \sqrt[3]{t}}{y^2}\right)$

b) $4\ln a - \left(\ln b - \frac{2}{3}\ln c\right)$

$\ln a^4 - \ln b + \frac{2}{3}\ln c$

$\ln c^{2/3}$

$\ln\left(\frac{a^4 \cdot \sqrt[3]{c^2}}{b}\right)$

c) $3(5\ln x + 2\ln y) + 2(\ln y - 7\ln x)$

$3(\ln x^5 + \ln y^2) + 2(\ln y - \ln x^7)$

$3\ln(x^5y^2) + 2\ln\left(\frac{y}{x^7}\right)$

$\ln(x^5y^2)^3 + \ln\left(\frac{y}{x^7}\right)^2$

$\ln\left(x^{15}y^6 \cdot \frac{y^2}{x^{14}}\right) = \ln(xy^8)$

6) Solve each of the following logarithmic or exponential equations:

a) $\log_3(x^2+8) - \log_3 4 = 3$

$\log_3 \frac{x^2+8}{4} = 3$

$3^3 = \frac{x^2+8}{4}$

$108 = x^2 + 8$

$100 = x^2$

$\boxed{x = \pm 10}$

b) $\ln(x+7) + \ln(x+3) = \ln 77$

$\ln(x^2+10x+21) = \ln 77$

$x^2 + 10x + 21 = 77$

$x^2 + 10x - 56 = 0$

$(x+14)(x-4) = 0$

$\cancel{x = -14}$ $\boxed{x = 4}$

extraneous

c) $\log_4(x^2-3) + \log_4 10 = 1$

$\log_4(10x^2 - 30) = 1$

$4^1 = 10x^2 - 30$

$10x^2 = 34$

$x^2 = \frac{34}{10} = \frac{17}{5}$

$\boxed{x = \pm\sqrt{\frac{17}{5}}}$

d) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$(6^{-1})^{3x+2} \cdot (6^3)^{3x} = 6^{-3}$

$6^{-3x-2} \cdot 6^{9x} = 6^{-3}$

$6^{6x-2} = 6^{-3}$

$6x - 2 = -3$

$6x = -1$

$\boxed{x = -\frac{1}{6}}$

e) $9^{-3x} \cdot 9^x = 27$

$9^{-2x} = 27$

$(3^2)^{-2x} = 3^3$

$3^{-4x} = 3^3$

$-4x = 3$

$\boxed{x = -\frac{3}{4}}$

f) $16^{2x-3} \cdot 4^{-2x} = 2^4$

$(2^4)^{2x-3} \cdot (2^2)^{-2x} = 2^4$

$2^{8x-12} \cdot 2^{-4x} = 2^4$

$2^{4x-12} = 2^4$

$4x - 12 = 4$

$4x = 16$

$\boxed{x = 4}$