Name: _____

AP Calculus AB Final Exam Review #1 NO CALCULATOR

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. If $\int_a^b f(x)dx = a + 2b$, then $\int_a^b \big(f(x) + 5\big)dx =$

   A. $5b - 5a$
   B. $a + 2b + 5$
   C. $7b - 5a$
   D. $7b - 6a$
   E. $7b - 4a$

2. $\frac{1}{2}\int e^{\frac{t}{2}}dt =$

   A. $e^{-t} + C$
   B. $e^{\frac{t}{2}} + C$
   C. $e^{-\frac{t}{2}} + C$
   D. $e^{t} + C$
   E. $2e^{\frac{t}{2}} + C$

3. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

   A. $\left(1, \frac{1}{2}\right)$
   B. $\left(\frac{1}{2}, \frac{1}{8}\right)$
   C. $\left(1, -\frac{1}{4}\right)$
   D. $\left(\frac{1}{2}, -\frac{1}{2}\right)$
   E. $(2,2)$

4. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

   A. $\frac{16\pi}{5}$
   B. π
   C. $\frac{16\pi}{3}$
   D. $\frac{8\pi}{3}$
   E. $\frac{32\pi}{5}$

5. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx$ is

   A. 0
   B. e + 1
   C. e - 1
   D. 1
   E. e

6. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4, 3)?

   A. $\frac{25}{27}$
   B. $\frac{7}{27}$
   C. $-\frac{7}{27}$
   D. $\frac{3}{4}$
   E. $-\frac{25}{27}$

7. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

A. $\dfrac{1}{\sqrt{2x-3}}$

B. $\dfrac{x}{\sqrt{2x-3}}$

C. $\dfrac{-x+3}{\sqrt{2x-3}}$

D. $\dfrac{3x-3}{\sqrt{2x-3}}$

E. $\dfrac{5x-6}{2\sqrt{2x-3}}$

8. $\dfrac{d}{dx}\cos^2(x^3) =$

A. $-2\sin(x^3)\cos(x^3)$

B. $6x^2\cos(x^3)$

C. $\sin^2(x^3)$

D. $6x^2\sin(x^3)\cos(x^3)$

E. $-6x^2\sin(x^3)\cos(x^3)$

9. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

A. $x < -2$ *or* $x > -\dfrac{2}{3}$

B. $x > 0$

C. $x < \dfrac{2}{3}$ *or* $x > 2$

D. $\dfrac{2}{3} < x < 2$

E. $x < 0$

10. $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

A. 0

B. 1

C. e

D. $\dfrac{1}{e}$

E. nonexistent

11. The expression $\dfrac{1}{50}\left(\sqrt{\dfrac{1}{50}} + \sqrt{\dfrac{2}{50}} + \sqrt{\dfrac{3}{50}} + \ldots + \sqrt{\dfrac{50}{50}}\right)$ is a Riemann sum approximation for

A. $\dfrac{1}{50}\int_0^{50} \sqrt{x}\,dx$

B. $\int_0^1 \sqrt{x}\,dx$

C. $\dfrac{1}{50}\int_0^1 \sqrt{x}\,dx$

D. $\int_0^1 \sqrt{\dfrac{x}{50}}\,dx$

E. $\dfrac{1}{50}\int_0^1 \sqrt{\dfrac{x}{50}}\,dx$

12. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

A. $\dfrac{16}{3}$

B. 8π

C. $\dfrac{32}{3}$

D. $\dfrac{14}{3}$

E. $\dfrac{28}{3}$

13. The average value of $\cos x$ on the interval [-3, 5] is

A. $\dfrac{\sin 3 + \sin 5}{2}$

B. $\dfrac{\sin 3 + \sin 5}{8}$

C. $\dfrac{\sin 3 - \sin 5}{2}$

D. $\dfrac{\sin 5 - \sin 3}{8}$

E. $\dfrac{\sin 5 - \sin 3}{2}$

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

A. 0.4
B. 0.5
C. 2.6
D. 3.4
E. 5.5

15. What are all of the values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

A. $x < -1$ *and* $x > 3$
B. $-1 < x < 3$
C. $-3 < x < 1$
D. There are no such value of x.
E. All values of x.

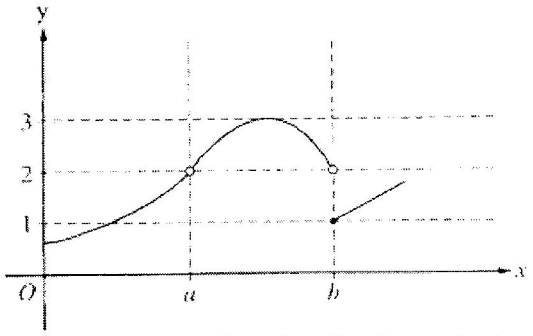
16. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

A. $y = 2\left(x - \frac{\pi}{4}\right)$
B. $y = -2\left(x - \frac{\pi}{4}\right)$
C. $y - 1 = -\left(x - \frac{\pi}{4}\right)$
D. $y - 1 = -2\left(x - \frac{\pi}{4}\right)$
E. $y = -\left(x - \frac{\pi}{4}\right)$

17. $\int_1^2 \left(4x^3 - 6x\right)dx =$

A. 2
B. 4
C. 6
D. 36
E. 42

18.



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

A. $\lim_{x \to a} f(x) = 2$
B. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$
C. $\lim_{x \to a} f(x)$ does not exist.
D. $\lim_{x \to b} f(x) = 2$
E. $\lim_{x \to b} f(x) = 1$

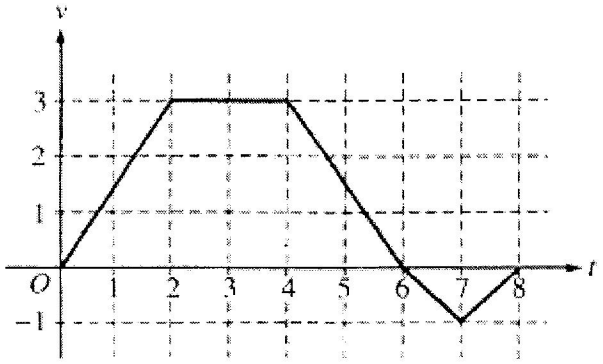
19. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

A. -3
B. 1
C. -1
D. 3
E. -5

20. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

A. $(2, \infty)$
B. $(-2, \infty)$
C. $(-2, 4)$
D. $(-\infty, 2)$
E. $(-\infty, \infty)$

Questions 21-22 refer to the following situation.



A bug begins to crawl up a vertical wire at time t = 0. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

21. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

A. 14
B. 13
C. 11
D. 8
E. 6

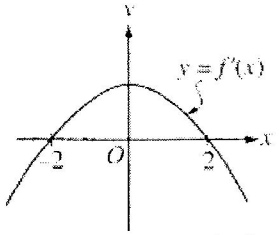
22. At what value of t does the bug change direction?

A. 2
B. 4
C. 6
D. 7
E. 8

23. Evaluate the following limit: $\lim_{x \to \infty} \dfrac{\sin\left(\frac{3}{x}\right)}{\frac{1}{x}}$

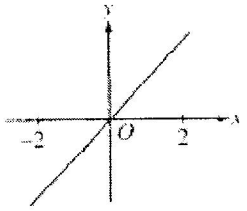
A. does not exist
B. -3
C. 3
D. 0
E. 1

24.

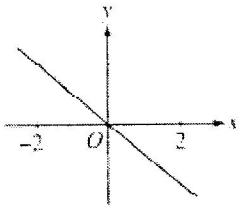


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

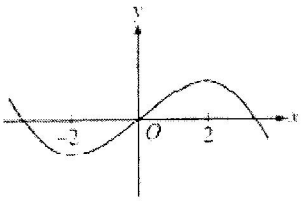
A.



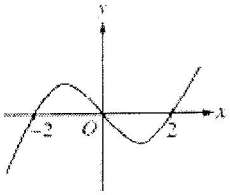
B.



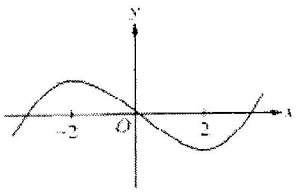
C.



D.



E.



AP Calculus AB Final Exam Review #1 CALCULATOR-ACTIVE

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$?

A. 8
B. 12
C. 16
D. 24
E. 32

2. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

A. 57.60
B. 57.88
C. 59.20
D. 60.00
E. 67.40

3. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

A. -0.46
B. 0.20
C. 0.91
D. 0.95
E. 3.73

4. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

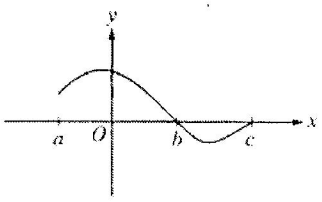
5. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then c =

A. 1
B. 4
C. $\dfrac{1}{4}$
D. $\dfrac{1}{\sqrt{2}}$
E. $\dfrac{1}{2\sqrt{2}}$

6. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

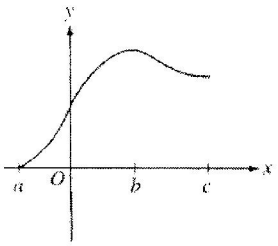
A. $\dfrac{2}{3}$
B. $\dfrac{1}{3}\left(e^3 - 1\right)$
C. $\dfrac{1}{2}$
D. 1
E. 2

7.

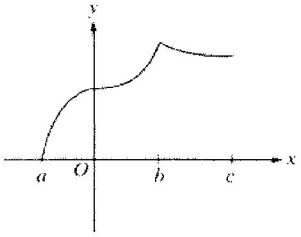


Let $f(x) = \int_a^x h(t)dt$, where h has the graph shown above. Which of the following could be the graph of f?

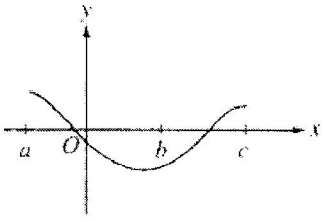
A.



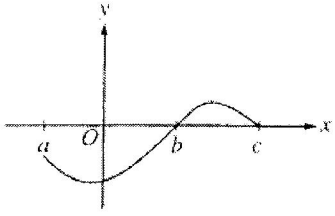
B.



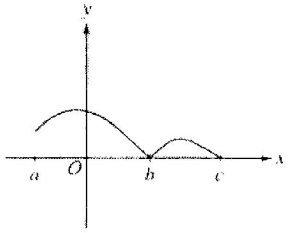
C.



D.



E.



8. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

A. 1

B. $\dfrac{e^{2x}(2x+1)}{2x^2}$

C. e^{2x}

D. $\dfrac{e^{2x}(2x+1)}{x^2}$

E. $\dfrac{e^{2x}(2x-1)}{2x^2}$

9. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

A. 1.02
B. 1.48
C. 1.85
D. 2.81
E. 3.14

10. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

A. 0.127
B. 0.385
C. 0.400
D. 0.600
E. 0.947

11. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at x =

A. -1.58
B. -1.63
C. -1.67
D. -1.89
E. -2.33

12. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at x = 2.
II. f is differentiable at x = 2.
III. The derivative of f is continuous at x = 2.

A. I only
B. II only
C. I and II only
D. I and III only
E. II and III only

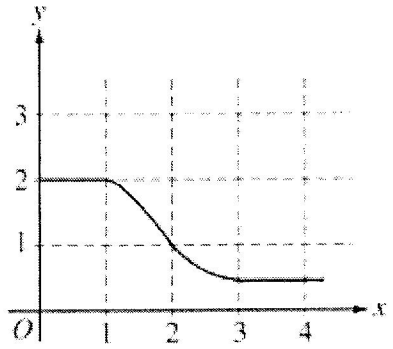
13. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

A. 0.168
B. 0.276
C. 0.318
D. 0.342
E. 0.551

14. If $f(x) = \ln|x^2 - 1|$, then $f'(x) =$

A. $\frac{1}{x^2-1}$
B. $\frac{2x}{|x^2-1|}$
C. $\frac{2x}{x^2-1}$
D. $\left|\frac{2x}{x^2-1}\right|$
E. $\frac{2|x|}{x^2-1}$

15.



The graph of f is shown in the figure above. If $\int_1^3 f(x)dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

A. 0.3
B. 1.3
C. 3.3
D. 4.3
E. 5

16. If $y = 2x - 8$, what is the minimum value of the product xy?

A. -4
B. 2
C. 0
D. -16
E. -8