NOTES--APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC EQUATIONS

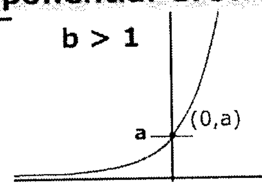
Simple Growth/Decay Model

$f(x) = a \cdot b^x$ a & b are positive

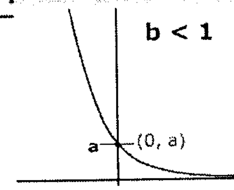
$b > 1 \rightarrow$ growth

$b < 1 \rightarrow$ decay

Exponential Growth



Exponential Decay



The Population Growth/Decay Model (use when the population changes at a constant rate)

$p(t) = p_0(1 + r)^t$

p_0 = initial population r = rate of change as a decimal t = time

Half-Life Model

"half-life" is the time it takes for a sample to be half of the amount it was before

$A(t) = A_0 \cdot e^{-kt}$

A_0 = initial amount k is a constant t = time

OR

$A(t) = A_0 \cdot (0.5)^{t/n}$,

A_0 = initial amount t = time n = the half life

Newton's Law of Cooling

$T(t) = T_m + (T_0 - T_m)e^{-kt}$

T_m = temperature of the medium T_0 = initial temperature of the object

t = time k is a constant

Chemical Acidity

$pH = -\log[H^+]$

H^+ = hydrogen-ion concentration

If pH < 7 → acid

If pH = 7 → neutral

If pH > 7 → base

Sound Intensity Model

$dB = 10 \log\left(\frac{I}{1 \times 10^{-12}}\right)$

I = intensity (in watts per square meter)

Finance Models

"compounded annually" $A = P(1 + r)^t$

"compounded in periods" $A = P(1 + \frac{r}{k})^{tk}$

"compounded continuously" $A = Pe^{rt}$

P = principal amount r = rate as a decimal t = time (years)

k = # times compounded per year

Example 1 Suppose that the half-life of a compound is 20 days with 5 grams present initially. Find the time when there will be 1 gram remaining.

n, A_0, t, A

$$A(t) = A_0 (0.5)^{t/n} \qquad 1 = 5(0.5)^{t/20} \qquad \boxed{t = 46.44 \text{ days}}$$

Example 2 Suppose you are given \$500 to invest. What annual interest rate compounded quarterly is required to double the money in 10 years?

$$A = P\left(1 + \frac{r}{k}\right)^{tk} \qquad 1000 = 500\left(1 + \frac{r}{4}\right)^{10 \cdot 4} \qquad r = .0699 \qquad \boxed{6.99\%}$$

Example 3 Given the data in the chart below, predict the panda population in 2020 for the Powell Zoo.

$x = 120$

year	population
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7

exp. regression

$$y = 80.075 * 1.013^x$$

$x = 120, \; y = 381.18621$

381 pandas

Example 4 Suppose a population in 1910 was 4200 and increased at 2.25% per year. Estimate the population in 1930 & 1945. Predict when the population reached 20,000.

P_0, $r = .0225$, $t = 20$, $t = 35$

$$p(t) = P_0 (1 + r)^t$$

1930: $p(20) = 4200(1 + .0225)^{20} = \boxed{6554}$

1945: $p(35) = 4200(1 + .0225)^{35} = \boxed{9150}$

$20000 = 4200(1 + .0225)^t$

$t = 70.14$ yrs

1980

Example 5 The noise level in a cafeteria is 125 dB. Find I.

$$dB = 10 \log\left(\frac{I}{1 \times 10^{-12}}\right) \qquad 125 = 10 \log\left(\frac{I}{1 \times 10^{-12}}\right)$$

$\boxed{I = 3.16 \text{ watts}/m^2}$

Example 6 A hard-boiled egg at 96°C is placed in 16°C water to cool. Four minutes later the temperature of the egg is 45°C. Determine when the egg will be 20°C.

$$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

$$45 = 16 + (96 - 16)e^{-k(4)}$$

$$k = .2536827$$

$$20 = 16 + 80e^{-.2536827t}$$

$\boxed{t = 11.81 \text{ minutes}}$