culus Name: ______________________

Notes 1.2 (Part 3)

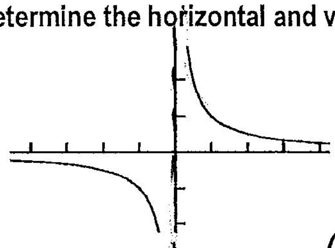
Goal #1: Students will be able to identify vertical and horizontal asymptotes.
Goal #2: Students will be able to state the end behavior of a function using limit notation.
Goal #3: Students will be able to determine if a function is continuous or discontinuous & the type.

Asymptotes

- An **ASYMPTOTE** is a line that the graph of the function approaches. As the graph of the function approaches $\pm\infty$ (or a real #), the function curve gets closer and closer to the asymptote without actually touching it.
  *Note: Sometimes a horizontal asymptote can be intersected by part of the graph.
- **Vertical Asymptotes**: The line $x = a$ is a vertical asymptote of a function $f(x)$ if f approaches + or $-\infty$ as x approaches "a" from the left or the right.
  *By finding any zeroes in the denominator, you will get the Vertical Asymptotes of the function.
- **Horizontal Asymptotes**: The line $y = b$ is a horizontal asymptote of a function $f(x)$ if f approaches "b" as x approaches + or $-\infty$
  *Finding Horizontal asymptotes →
  Option 1) When the degree of the denominator is $>$ than the degree of the numerator, the horizontal asymptote will be at $y = 0$. $\frac{\text{smaller}}{\text{bigger}}$ bottom heavy
  Option 2) When the degree of the numerator is $=$ to the degree of the denominator, the horizontal asymptote will be at: $y = \frac{leading\ coefficient\ of\ the\ numerator}{leading\ coefficient\ of\ the\ denominator}$ Example: $y = \frac{6x^2+3}{2x^2+x}$ has a H.A. at $y = \frac{6}{2} = 3$
  Option 3) When the degree of the numerator is $>$ than the degree of the denominator, there is no horizontal asymptote but a Oblique/slant asymptote. $\frac{\text{bigger}}{\text{smaller}}$ top heavy ∞ or $-\infty$

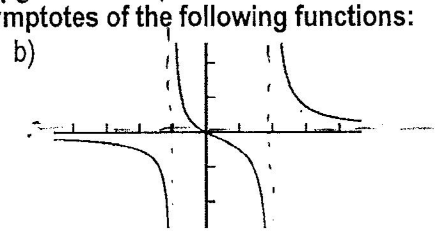
Determine the horizontal and vertical asymptotes of the following functions:

a)



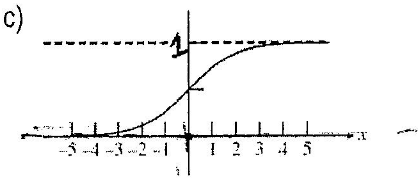
Horizontal Asymptote: $y = 0$
Vertical Asymptote(s): $x = 0$

b)



Horizontal Asymptote: $y = 0$
Vertical Asymptote(s): $x = -1$ $x = 2$

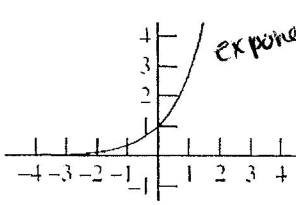
c)



Horizontal Asymptotes: $y = 1$ $y = 0$
Vertical Asymptote(s): none

NOW YOU TRY

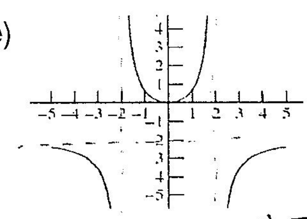
d)



exponential

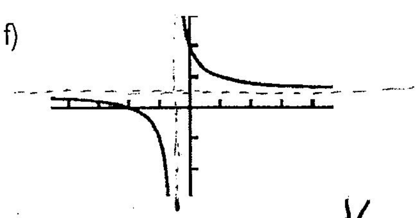
Horizontal Asymptote: $y = 0$
Vertical Asymptote(s): none

e)



Horizontal Asymptote: $y = -2$
Vertical Asymptote(s): $x = -2$, $x = 2$

f)



Horizontal Asymptote: $y = \frac{1}{2}$
Vertical Asymptote(s): $x = -\frac{1}{2}$

g) $y = \frac{4x^2+4x}{2x^2-2} = \frac{4x(x+1)}{2(x^2-1)}$
$= \frac{4x(x+1)}{2(x+1)(x-1)}$ hole at $x=-1$

Horizontal Asymptote: $y = \frac{4}{2} = 2$
Vertical Asymptote(s): $x = 1$

h) $y = \frac{2x-3}{x^2+3x-4} = \frac{2x-3}{(x+4)(x-1)}$

Horizontal Asymptote: $y = 0$
Vertical Asymptote(s): $x = -4$ $x = 1$

i) $y = \frac{x^2+4}{x+3}$

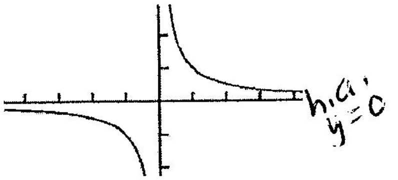
Horizontal Asymptote: none
Vertical Asymptote(s): $x = -3$

End Behavior

- **Left End Behavior (LEB)**: The left end behavior of a graph of the function $f(x)$ describes the behavior of $f(x)$ as $x \to -\infty$
- **Right End Behavior (REB)**: The right end behavior of a graph of the function $f(x)$ describes the behavior of $f(x)$ as $x \to \infty$

State the left and right end behavior of the graphs below using limit notation:

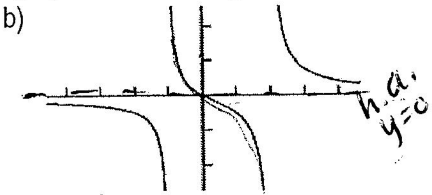
a)



LEB: $\lim_{x \to -\infty} f(x) = 0$

REB: $\lim_{x \to \infty} f(x) = 0$

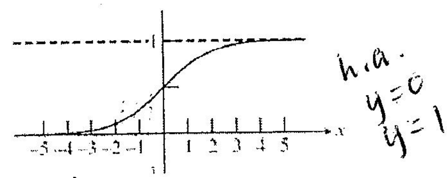
b)



LEB: $\lim_{x \to -\infty} f(x) = 0$

REB: $\lim_{x \to \infty} f(x) = 0$

c)

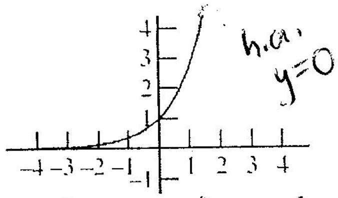


LEB: $\lim_{x \to -\infty} f(x) = 0$

REB: $\lim_{x \to \infty} f(x) = 1$

NOW YOU TRY

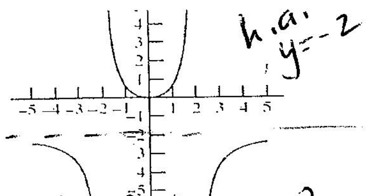
d)



LEB: $\lim_{x \to -\infty} f(x) = 0$

REB: $\lim_{x \to \infty} f(x) = \infty$

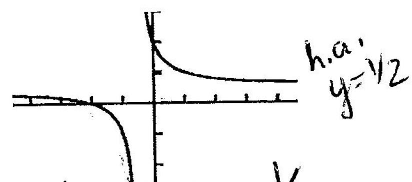
e)



LEB: $\lim_{x \to -\infty} f(x) = -2$

REB: $\lim_{x \to \infty} f(x) = -2$

f)

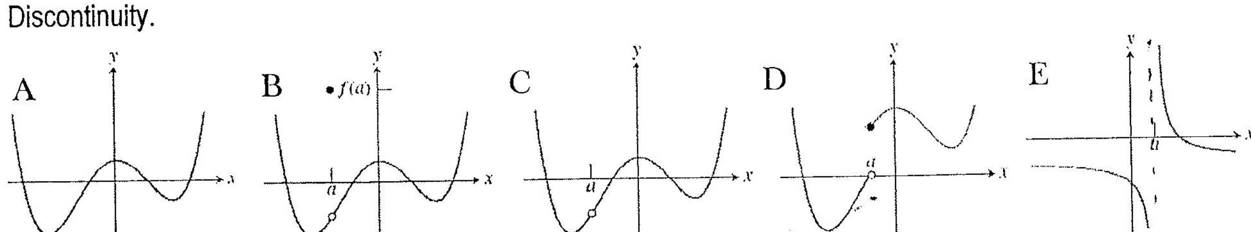


LEB: $\lim_{x \to -\infty} f(x) = \frac{1}{2}$

REB: $\lim_{x \to \infty} f(x) = \frac{1}{2}$

Discontinuity

- **Continuous Function:** A function where the graph does not come apart at any point on its domain.
- If a function is not continuous, then it could have one of the following: Removable Discontinuity, Jump Discontinuity, or Infinite Discontinuity.



Continuous at all x | Removable discontinuity | Removable discontinuity | Jump discontinuity | Infinite discontinuity

NOTE: Jump and infinite discontinuities are also referred to as "Non-Removable"

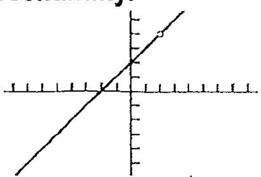
A) Continuous: can trace & not pick up pencil

B and C) Removable Discontinuity: hole

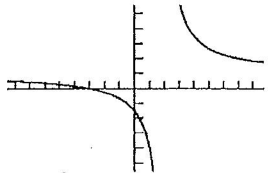
D) Jump Discontinuity: pick up pencil & jump elsewhere

E) Infinite Discontinuity: vertical asymptote

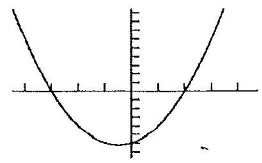
...e the continuity of each of the following functions. If the function is discontinuous state whether the discontinuity is ...able or non-removable. If there is a non-removable discontinuity in the graph state if it has jump or infinite ...continuity.

a)


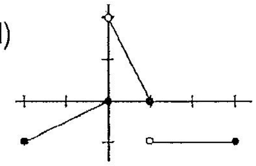
discont.
removable (hole)

b)


discont.
infinite

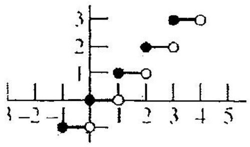
c)


cont.

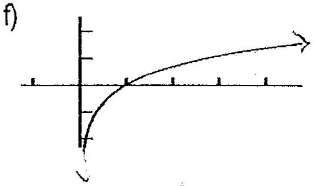
d)


discont.
jump

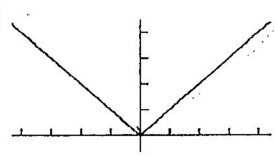
NOW YOU TRY

e)


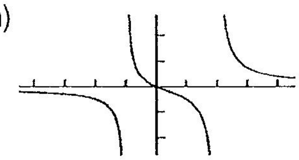
discont.
jump

f)


cont

g)


cont.

h)


discont.
infinite