Building a New Series from a Known Series

Things to know:

- Known series are Maclaurin. You can't use them to build a series centered elsewhere.
- the I.O.C. only changes if you do calculus. (endpts), not if arithmetic or algebra.
- Replace first, then do multiplication

EX1 Find the Maclaurin series.

a) $f(x) = \frac{1}{1+x^2}$

know $\frac{1}{1-x} = \sum x^n$

build $\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum (-x)^n = \sum (-1)^n x^n$

$$\frac{1}{1+x^2} = \sum (-1)^n (x^2)^n = \sum (-1)^n x^{2n}$$

b) $f(x) = x^3 \cdot e^{-4x}$

know $e^x = \sum \frac{x^n}{n!}$

$$e^{-4x} = \sum \frac{(-4x)^n}{n!} = \sum \frac{(-4)^n x^n}{n!}$$

$$x^3 \cdot e^{-4x} = \sum \frac{(-4)^n x^n}{n!} \cdot x^3 = \sum \frac{(-4)^n x^{n+3}}{n!}$$

C) $f(x) = \dfrac{1}{(1-x)^2}$

Know $\dfrac{1}{1-x} = \sum x^n$ deriv: $\sum n \cdot x^{n-1}$

$\dfrac{1}{1-x} = (1-x)^{-1}$

deriv: $-1(1-x)^{-2}(-1) = \dfrac{1}{(1-x)^2}$

D) $f(x) = \ln(1+x)$

know $\dfrac{1}{1-x} = \sum x^n$

build $\dfrac{1}{1+x} = \sum (-1)^n \cdot x^n$

$\int \dfrac{1}{1+x} dx = \sum \dfrac{(-1)^n x^{n+1}}{n+1} + C$

EX 2 Find the 1st 3 terms of the maclaurin series for $f(x)$.

A. $f(x) = e^{2x} \cos 3x$

know $e^x = \sum \dfrac{x^n}{n!}$

$e^{2x} = \sum \dfrac{(2x)^n}{n!}$

know $\cos x = \sum \dfrac{(-1)^n x^{2n}}{(2n)!}$

$$\cos(3x) = \sum \frac{(-1)^n (3x)^{2n}}{(2n)!}$$

$$e^{2x}\cos 3x = \left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \ldots\right)\left(1 - \frac{9}{2}x^2 + \frac{81}{24}x^4 - \ldots\right)$$

$$\frac{81}{24} \downarrow \frac{27}{8}$$

$$= 1 - \frac{9}{2}x^2 + \frac{27}{8}x^4 - \ldots + 2x - 9x^3 + \frac{27}{4}x^5 + \ldots$$

$$+ 2x^2 - 9x^4 + \ldots$$

$$= \boxed{1 + 2x - \frac{5}{2}x^2}$$

B. $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \;\overline{)\; x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots}$$

$$= x + \frac{1}{3}x^3 + \frac{2}{15}x^5$$

$$- \left(x - \frac{x^3}{2!} + \frac{x^5}{4!} - \ldots\right)$$

$$\frac{1}{3}x^3 - \frac{1}{30}x^5 - \ldots$$

$$- \left(\frac{1}{3}x^3 - \frac{1}{6}x^5 + \ldots\right)$$

$$\frac{2}{15}x^5 + \ldots$$

$$-\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$$

$$\frac{1}{120} - \frac{1}{24} = -\frac{1}{30}$$

$$-\frac{1}{30} + \frac{1}{6} = \frac{2}{15}$$

why?

Find $\int e^{x^2} dx$

start $e^x = \sum \frac{x^n}{n!}$

build $e^{x^2} = \sum \frac{(x^2)^n}{n!}$

then integrate ↑

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