AP Calculus BC — Practice 3.5: Worksheet on Building Series

Work the following on notebook paper without a calculator.

1. Find the Taylor series for $f(x) = e^x$ centered at $x = 3$.
2. Find the Maclaurin series for $f(x) = e^{5x}$.
3. Find the Taylor series for $f(x) = \sin x$ centered at $x = \pi/2$.
4. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = \cos(\pi x)$.
5. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = e^{-x/2}$.
6. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = x^2 e^{-x}$.
7. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = \dfrac{\sin x}{x}$. Use the series that you obtain to evaluate the indefinite integral $\displaystyle\int \frac{\sin x}{x}\,dx$ as an infinite series.

8. Find the Taylor polynomial of the given degree for the given function, centered at the given point. (Leave your answers as sums of powers of $(x - a)$).

   a) $f(x) = \ln x$, degree 3, centered at $x = 2$.
   b) $f(x) = \sqrt{1+x}$, degree 3, centered at $x = 0$.
   c) $f(x) = \sin x$, degree 3, centered at $\pi/3$.
   d) $f(x) = \cos x$, degree 4, centered at $x = \pi$.

Answers:

1. $\sum \frac{e^3}{n!}(x-3)^n$
2. $\sum \frac{(5x)^n}{n!}$
3. $\sum \frac{(-1)^n (x-\pi/2)^{2n}}{(2n)!}$
4. $\sum \frac{(-1)^n (\pi x)^{2n}}{(2n)!}$
5. $\sum \frac{(-1)^n x^n}{2^n \cdot n!}$ 8
6. $\sum \frac{(-1)^n x^{n+2}}{n!}$
7. $\sum \frac{(-1)^n x^{2n+1}}{(2n+1)!\,(2n+1)} + C$
8. a) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3$
   b) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$
   c) $\frac{\sqrt{3}}{2} + \frac{1}{2}\left(x-\frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(x-\frac{\pi}{3}\right)^2 - \frac{1}{12}\left(x-\pi/3\right.$
   d) $-1 + \frac{1}{2}(x-\pi)^2 - \frac{1}{24}(x-\pi)^4$