Name: ____________________

Notes 1.4 (Part 1)

Goal #1: Students will be able to find the domain of combinations AND compositions of functions.
Goal #2: Students will be able to decompose a function into two functions (neither of which is the identity function).
Goal #3: Students will be able identify implicitly defined functions within relations.

When dealing with ***SIMPLE*** combinations of functions (such as $f+g$, $f-g$, $f \bullet g$, and, $f \div g$) the domain of the function consists of all numbers that belong to BOTH the domain of f and the domain of g.

Given that $f(x)=\sqrt{2-x}$ & $g(x)=\sqrt{2x+5}$ determine the value of $f+g$, $f-g$, $f \bullet g$, and, $f \div g$ and their domains

$(f+g)(x) = f(x) + g(x)$ $\quad$ D: $[-5/2, 2]$

$= \sqrt{2-x} + \sqrt{2x+5}$

f: $2-x \geq 0$, $-x \geq -2$, $x \leq 2$ $\quad$ g: $2x+5 \geq 0$, $x \geq -\frac{5}{2}$

$(f-g)(x) = f(x) - g(x)$

$= \sqrt{2-x} - \sqrt{2x+5}$

D: $[-5/2, 2]$

$(f \bullet g)(x) = f(x) \bullet g(x)$

$= \sqrt{2-x} \bullet \sqrt{2x+5}$

$= \sqrt{(2-x)(2x+5)}$

$4x + 10 - 2x^2 - 5x$

$= \sqrt{-2x^2 - x + 10}$

D: $[-5/2, 2]$

$(f \div g)(x) = \frac{f(x)}{g(x)}$

$= \frac{\sqrt{2-x}}{\sqrt{2x+5}}$

D: $(-5/2, 2]$

Now you try:
Perform the indicated operation, simplify your result as much as possible, then determine the domain of the resulting function.

a) If $f(x)=\frac{4}{x-2}$ and $g(x)=\frac{-x^2}{x-2}$, find $(f+g)(x)$

$f(x) + g(x) = \frac{4}{x-2} + \frac{-x^2}{x-2}$

$= \frac{4-x^2}{x-2}$

f: $(-\infty,2)\cup(2,\infty)$

g: $(-\infty,2)\cup(2,\infty)$

D: $(-\infty,2)\cup(2,\infty)$

b) If $f(x)=\sqrt{x-10}$ and $g(x)=\sqrt{x+10}$, find $(fg)(x)$

$f(x) \bullet g(x)$

$\sqrt{x-10} \bullet \sqrt{x+10}$

$\sqrt{(x-10)(x+10)}$

$x^2 + 10x - 10x - 100$

$\sqrt{x^2-100}$

f: $x \geq 10$

g: $x \geq -10$

-10 $\quad$ 10

D: $[10, \infty)$

c) If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{x+3}$, find $(fg)(x)$

d) If $f(x)=\sqrt{x-5}$ and $g(x)=\sqrt{x+2}$, find $(f/g)(x)$

$\frac{f(x)}{g(x)} = \frac{\sqrt{x-5}}{\sqrt{x+2}}$

$x \neq -2$

f: $[5,\infty)$

g: $[-2,\infty)$

D: $[5,\infty)$

$f(g(x))$ "f of g of x"

When dealing with **COMPOSITIONS** functions (such as $f \circ g$) the domain of the function consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.

y-values

Ex2.a) Given $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$ Find the composition function and its domain.

a) $f(g(x)) = \sqrt{x^2+1-1} = \sqrt{x^2} = |x|$

$g: (-\infty, \infty)$ $D: (-\infty, \infty)$

b) $g(f(x)) = (\sqrt{x-1})^2 + 1$

$= x - 1 + 1$ $f: [1, \infty)$

$= x$ $D: [1, \infty)$

Ex2.b) Given $f(x) = 9 - x^2$ and $g(x) = \sqrt{x}$ Find the composition function and its domain.

a) $f(g(x)) = 9 - (\sqrt{x})^2 = 9 - x$

$g: [0, \infty)$ $D: [0, \infty)$

b) $g(f(x)) = \sqrt{9-x^2}$ $D: [-3, 3]$

$f: (-\infty, \infty)$ $9 - x^2 \geq 0$

$(3+x)(3-x) \geq 0$

$- \; + \; -$; -3, 3

Now you try:

2c) Given $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$ Find the composition function and its domain.

a) $f(g(x)) = (\sqrt{x})^2 - 1 = x - 1$

$g: [0, \infty)$ $D: [0, \infty)$

b) $g(f(x)) = \sqrt{x^2 - 1}$ $D: (-\infty, -1] \cup [1, \infty)$

$f: (-\infty, \infty)$ $x^2 - 1 \geq 0$

$(x+1)(x-1) \geq 0$

$+ \; - \; +$; -1, 1

2d) Given $f(x) = \dfrac{1}{x+3}$ and $g(x) = x^2 - 3$ Find the composition function and its domain.

a) $f(g(x)) = \dfrac{1}{x^2-3+3} = \dfrac{1}{x^2}$

$g: (-\infty, \infty)$ $D: (-\infty, 0) \cup (0, \infty)$

b) $g(f(x)) = \left(\dfrac{1}{x+3}\right)^2 - 3$ $D: (-\infty, -3) \cup (-3, \infty)$

$f: (-\infty, -3) \cup (-3, \infty)$

When **DECOMPOSING** functions the purpose is to create two functions (not using the identity function) such that their composition IS the given function. So, when given $h(x)$ the goal is to **DEFINE** $f(x)$ **&** $g(x)$ so that $h(x) = f(g(x))$

Ex3) For each of the following $h(x)$, find the functions f and g such that $h(x) = f(g(x))$.

a) $h(x) = (x+1)^2 - 3(x+1) + 4$

$f(x) = x^2 - 3x + 4$

$g(x) = x + 1$

b) $h(x) = \sqrt{x^3 + 1}$

$f(x) = \sqrt{x}$ OR $f(x) = \sqrt{x+1}$

$g(x) = x^3 + 1$ $g(x) = x^3$

c) $h(x) = 9x^2 + 6x - 2$

$h(x) = 3(3x^2 + 2x) - 2$

$f(x) = 3x - 2$

$g(x) = 3x^2 + 2x$

Now You Try ☺

d) $h(x) = \sqrt[3]{2x+1}$

$f(x) = \sqrt[3]{x}$ OR $f(x) = \sqrt[3]{x+1}$

$g(x) = 2x + 1$ $g(x) = 2x$

e) $h(x) = \dfrac{x+2}{(x+2)^2 + 1}$

$f(x) = \dfrac{x}{x^2+1}$

$g(x) = x + 2$

f) $h(x) = \dfrac{x+5}{x^2 + 10x + 25}$

g) $h(x) = \sqrt{\dfrac{1}{x}}$