Now You Try:

4) $\frac{x+2}{4x}+\frac{x}{3x^2+9x}$

5) $\frac{2x}{x^2-1}-\frac{x+2}{x^2+2x+1}$

*6) $\frac{3}{x}-\frac{5x}{x^3+1}+\frac{1}{x^2-1}$

$\frac{3x^2+19x+18}{12x(x+3)}$

$x \neq 0, -3$

$\frac{x^2+x+2}{(x+1)^2(x-1)}$

$x \neq 1, -1$

$\frac{3x^4-7x^3+4x^2+4x-3}{x(x+1)(x-1)(x^2-x+1)}$

$x \neq \pm 1, 0$

SIMPLIFYING COMPOUND FRACTIONS

The easiest way to work with compound fractions is to clearly identify a "top" and "bottom" and simplify what is on the "top" as if it were ITS OWN PROBLEM, meanwhile you will do the same thing with the "bottom." AFTER you have finished whatever must be done on the top & bottom, THEN you MULTIPLY BY THE RECIPROCAL

Ex16) $\dfrac{3-\frac{7}{X+2}}{1-\frac{1}{X-3}}$ LCD = x+2, LCD = x−3

$\frac{3(x+2)-7}{x+2}=\frac{3x+6-7}{x+2}=\frac{3x-1}{x+2}$

$\frac{1(x-3)-1}{x-3}=\frac{x-3-1}{x-3}=\frac{x-4}{x-3}$

$\dfrac{\frac{3x-1}{x+2}}{\frac{x-4}{x-3}}=\frac{3x-1}{x+2}\cdot\frac{x-3}{x-4}=\boxed{\frac{(3x-1)(x-3)}{(x+2)(x-4)}}$

$x \neq -2, 3, 4$

Ex17) $\dfrac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a}-\frac{1}{b}}$ LCD = a^2b^2, LCD = ab

$\frac{b^2-a^2}{a^2b^2}$

$\frac{b-a}{ab}$

$\dfrac{\frac{b^2-a^2}{a^2b^2}}{\frac{b-a}{ab}}=\frac{(\cancel{b-a})(b+a)}{a^{\cancel{2}}b^{\cancel{2}}}\cdot\frac{\cancel{ab}}{\cancel{b-a}}=\boxed{\frac{b+a}{ab}}$

$a \neq 0$, $b \neq 0$, $b \neq a$

Now You Try

7) $\dfrac{\frac{3}{x+1}}{\frac{3}{x+1}-\frac{1}{x}}$ LCD = $x(x+1)$

$\frac{3x-1(x+1)}{x(x+1)}=\frac{3x-x-1}{x(x+1)}=\frac{2x-1}{x(x+1)}$

$\dfrac{\frac{3}{x+1}}{\frac{2x-1}{x(x+1)}}=\frac{3}{\cancel{x+1}}\cdot\frac{x(\cancel{x+1})}{2x-1}=\boxed{\frac{3x}{2x-1}}$

$x \neq -1, 0, \frac{1}{2}$

8) $\dfrac{\frac{x-2}{3x+1}}{\frac{1}{x}-\frac{2}{3x+1}}$ LCD = $x(3x+1)$

$\frac{1(3x+1)-2x}{x(3x+1)}=\frac{x+1}{x(3x+1)}$

$\dfrac{\frac{x-2}{3x+1}}{\frac{x+1}{x(3x+1)}}=\frac{x-2}{\cancel{3x+1}}\cdot\frac{x(\cancel{3x+1})}{x+1}=\frac{x(x-2)}{x+1}$

4

$x \neq -\frac{1}{3}, 0, -1$

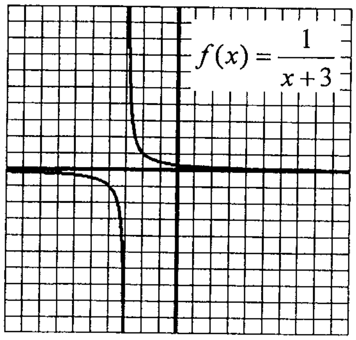
Precalculus   Name: ______________________

2.7: Graphs of Rational Functions

Definition-------RATIONAL FUNCTION: Let $f(x)$ & $g(x)$ be polynomials with $g(x) \neq 0$. Then $h(x) = \frac{f(x)}{g(x)}$ is a "rational function."

*****__Note__: The domain of the function $h(x)$ in the above definition is the set of all real numbers except the zeros of $g(x)$*****

Ex1) Find the domain of f and use limits to describe its behavior at values of x not on its domain.



D: $(-\infty, -3) \cup (-3, \infty)$

parent $y = \frac{1}{x}$
shift left 3

$\lim_{x \to -3^-} f(x) = -\infty$ from the left

$\lim_{x \to -3^+} f(x) = \infty$ from the right

$\lim_{x \to -3} f(x) = \text{DNE}$

V.A. $x = -3$
H.A. $y = 0$

Is there anything familiar about this graph? Could you have figured out how to Sketch it using transformations of a function we know?

****Graphs of Rational Functions****

***END BEHAVIOR ASYMPTOTES*:**

Situation #1--- degree numer < degree denom "bottom heavy"

H.A. $y = 0$

Example: $y = \frac{x^2-3x+5}{2x^3-1}$

Situation #2--- degree numer. = deg. denom.

Example: $y = \frac{3x^2-2x+7}{2x^2-1}$ H.A. $y = \frac{\text{l.c. numer.}}{\text{l.c. denom}}$ $y = \frac{3}{2}$

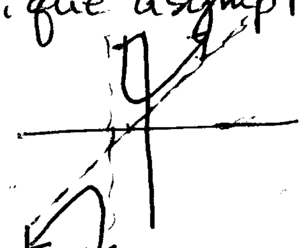
Situation #3--- degree numer. > degree denom. "top heavy"

Example: $y = \frac{x^2+3x+5}{x+2}$

no H.A.
slant/oblique asymptote
$y = x + 1$

$$\begin{array}{r|rrr} -2 & 1 & 3 & 5 \\ & & -2 & -2 \\ \hline & 1_x & 1_{const} & 3_{rem} \end{array}$$

ignore



Ex2) Find the end behavior asymptotes of the following rational functions:

a) $y = \frac{5x^2-x+9}{2x^2-2x+4}$ H.A. $y = \frac{5}{2}$

b) $y = \frac{x^4}{2x^7-1}$ H.A. $y = 0$

c) $y = \frac{x^3+2x^2-x+5}{x-1}$

no H.A.
oblique asymptote $y = x^2 + 3x + 2$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & 5 \\ & & 1 & 3 & 2 \\ \hline & 1_{x^2} & 3_x & 2_{const.} & 7_{rem} \end{array}$$

VERTICAL ASYMPTOTES: occur at the zeros of the denom. after simplifying the rational function

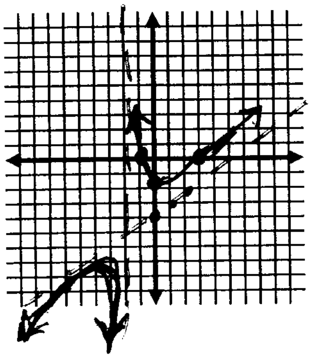
REMOVABLE DISCONTINUITIES: "holes" – happens when a factor is divided out. location is $(x, f(x))$ found after simplifying

X-INTERCEPTS: Zeros of numerator after simplifying
(#, 0)

Y-INTERCEPTS: value $f(0)$, if defined
(0, #)

Ex3) Determine asymptotes and/or holes of the functions below, along with intercepts. Then, sketch the graph, & write behavior statements for REB, LEB, & both sides of any vertical asymptotes (VA).

A) $f(x) = \dfrac{x^2 - 2x - 3}{x+2}$



Holes: NONE

x-ints: $(3,0)$ $(-1,0)$

y-int: $(0, -3/2)$

VA: $x = -2$

End Behavior Asymptote: $y = x - 4$ (slant)

$f(x) = \dfrac{(x-3)(x+1)}{x+2}$ $\quad f(0) = \dfrac{0^2 - 2(0) - 3}{0+2} = -\dfrac{3}{2}$

Statements:

$\lim\limits_{x\to-\infty} f(x) = -\infty$ $\quad \lim\limits_{x\to\infty} f(x) = \infty$

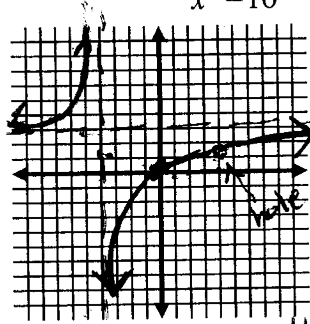
$\lim\limits_{x\to-2^+} f(x) = \infty$ $\quad \lim\limits_{x\to-2^-} f(x) = -\infty$

$\lim\limits_{x\to-2} f(x) =$ does not exist

$$\begin{array}{r|rrr} -2 & 1 & -2 & -3 \\ & \downarrow & -2 & 8 \\ \hline & 1_x & -4_{\text{const.}} & 5_{\text{rem.}} \end{array}$$

$y = x - 4$

B) $f(x) = \dfrac{3x^2 - 11x - 4}{x^2 - 16}$



Holes: $(4, 13/8)$

x-ints: $(-\frac{1}{3}, 0)$

y-int: $(0, \frac{1}{4})$

VA: $x = -4$

End Behavior Asymptote: H.A. $y = 3$

point $(-5, 14)$

$\dfrac{3(-5)+1}{-5+4} = \dfrac{-14}{-1}$

$f(x) = \dfrac{(3x+1)\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \dfrac{3x+1}{x+4}$

Statements:

$\lim\limits_{x\to-\infty} f(x) = 3$ $\quad \lim\limits_{x\to\infty} f(x) = 3$

$\lim\limits_{x\to-4^-} f(x) = \infty$ $\quad \lim\limits_{x\to4^-} f(x) = 13/8$

$\lim\limits_{x\to-4^+} f(x) = -\infty$ $\quad \lim\limits_{x\to4^+} f(x) = 13/8$

$\lim\limits_{x\to-4} f(x) =$ DNE $\quad \lim\limits_{x\to4} f(x) = 13/8$

$f(4) = \dfrac{3(4)+1}{4+4} = \dfrac{13}{8}$

$f(0) = \dfrac{3(0)+1}{0+4} = \dfrac{1}{4}$