

Notes (4.3)—Trigonometric Functions of Any Angle

- ✘ In Trigonometry we look at an angle in terms of a rotating ray. The beginning position of the ray is called the initial side of the angle.
- ✘ The ray is rotated about its end point called the vertex and the final position of the ray is called the terminal side of the angle.
- ✘ The degree of an angle is a number that describes the amount of rotation from the initial side to the terminal side of the angle.

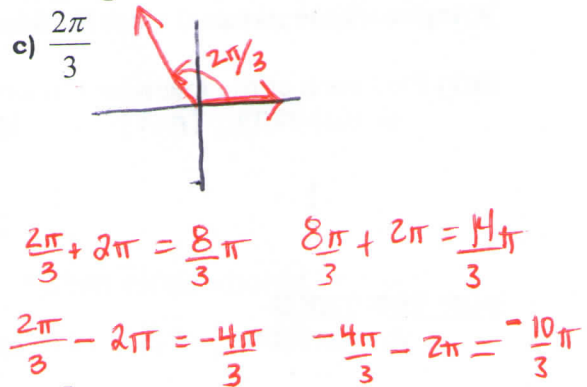
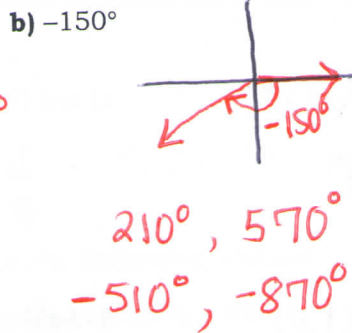
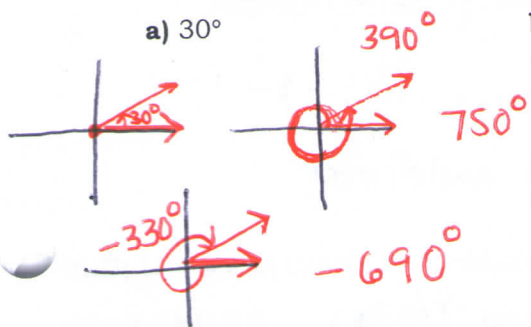
Positive angles are generated by counterclockwise rotations
 Negative angles are generated by clockwise rotations



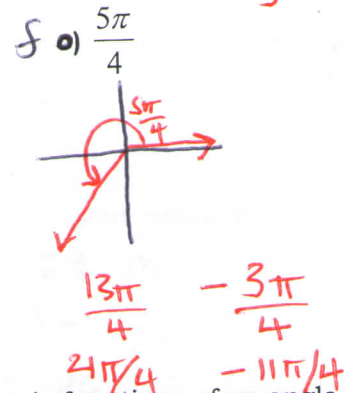
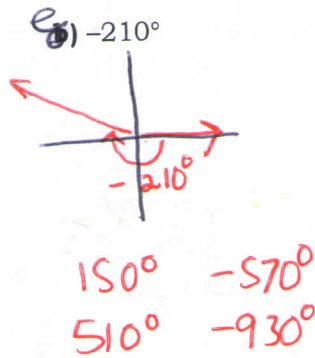
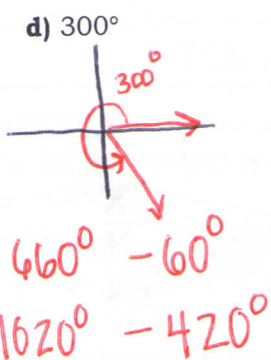
***NOTE: Typically angles are drawn in STANDARD POSITION with vertex at the origin & initial side on the positive x-axis.

- ✘ Because it is possible for two angles to have the same initial side and terminal side but different angle measures we refer to these angles as coterminal angles.

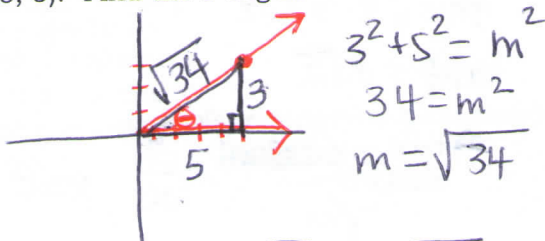
Ex1) Find & draw 2 positive and 2 negative angles that are co-terminal with the given angle.



NOW YOU TRY ☺

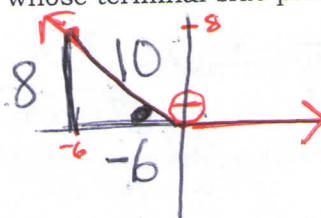


Ex2) Let θ be the acute angle in standard position whose terminal side contains (5, 3). Find the 6 trig functions of θ .



$\sin\theta = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$ $\csc\theta = \frac{\sqrt{34}}{3}$
 $\cos\theta = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$ $\sec\theta = \frac{\sqrt{34}}{5}$
 $\tan\theta = \frac{3}{5}$ $\cot\theta = \frac{5}{3}$

Ex3) Find the six trig functions of an angle whose terminal side passes through (-6, 8)



$(-6)^2 + 8^2 = m^2$
 $100 = m^2$
 $m = 10$

$\sin\theta = \frac{8}{10} = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$
 $\cos\theta = \frac{-6}{10} = -\frac{3}{5}$ $\sec\theta = -\frac{5}{3}$
 $\tan\theta = \frac{8}{-6} = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$

* A function $y=f(x)$ is _____ if there is a positive number c such that $f(t+c)=f(t)$ for all values of t in the domain of f . The smallest such number c is called the _____ of the function.

Ex4) Find each of the following (WITHOUT a calculator)

a) $\sin\left(\frac{57,801\pi}{2}\right)$

b) $\cos(288.45\pi) - \cos(280.45\pi)$

c) $\tan\left(\frac{\pi}{4} - 99,999\pi\right)$

sin \rightarrow "y" opp.
cos \rightarrow "x" adj.

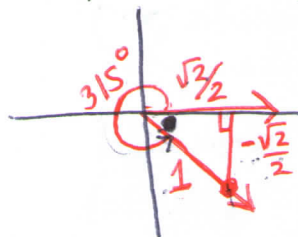
Ex5) Find the six trig functions of 315°

Ex6) Find each of the following

a) $\sin(-210^\circ)$

b) $\tan(5\pi/3)$

c) $\sec(-3\pi/4)$



$\sin 315^\circ = -\frac{\sqrt{2}}{2}$
 $\cos 315^\circ = \frac{\sqrt{2}}{2}$
 $\tan 315^\circ = -1$

$\csc 315^\circ = -\frac{2}{\sqrt{2}} = -\sqrt{2}$
 $\sec 315^\circ = \frac{2}{\sqrt{2}} = \sqrt{2}$
 $\cot 315^\circ = -1$

$\frac{1}{2}$
 $\frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

$\cos\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$-\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

* Angles whose terminal sides lie along one of the coordinate axes are called quadrantal angles.

Ex7) Find each of the following if it exists:

a) $\sin(-270^\circ)$ (0,1)

b) $\tan 3\pi$ (-1,0)

c) $\sec(11\pi/2)$ (0,-1)

1

$\frac{0}{-1} = 0$

$\frac{1}{0}$ undefined

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d) $\cos(180^\circ)$ (-1,0)

e) $\cot(0^\circ)$ (1,0)

f) $\csc(-\pi/2)$ (0,-1)

-1

$\frac{1}{0}$ undefined

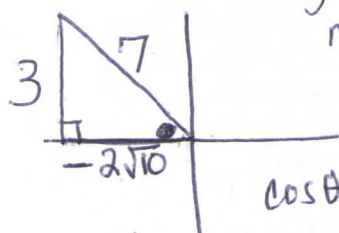
$\frac{1}{-1} = -1$

Ex8) Find $\cos\theta$ and $\tan\theta$ using the given information to construct a reference angle.

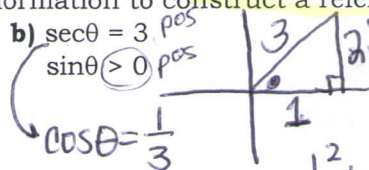
a) $\sin\theta = 3/7$ pos
 $\tan\theta < 0$ neg } quad 2

b) $\sec\theta = 3$ pos
 $\sin\theta > 0$ pos } quad 1

c) $\cot\theta$ is undefined
 $\sec\theta < 0$ neg } quad 2



$m^2 + 3^2 = 7^2$
 $m^2 = 40$
 $m = -\sqrt{40} = -2\sqrt{10}$
 $\cos\theta = \frac{-2\sqrt{10}}{7}$ $\tan\theta = \frac{3\sqrt{10}}{-2\sqrt{10}\sqrt{10}} = \frac{3\sqrt{10}}{-20}$



$\cos\theta = \frac{1}{3}$

$1^2 + m^2 = 3^2$

$m^2 = 8$

$m = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = 2\sqrt{2}$

$\frac{x}{y}$ $y=0$
at (-1,0)
 $\cos\theta = -1$
 $\tan\theta = \frac{0}{-1} = 0$

NOW YOU TRY ☺ Find $\sin\theta$ and $\cot\theta$ using the given information to construct a reference angle.

d) $\cos\theta = -12/13$
 $\sin\theta < 0$

e) $\csc\theta = -5$
 $\tan\theta < 0$

$\frac{y}{x}$ $x=0$
at (0,1)

$\sin\theta = 1$

$\cot\theta = \frac{0}{1} = 0$