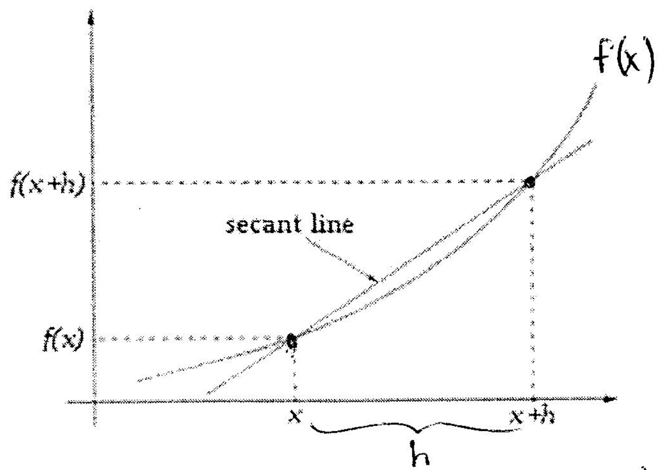
Notes--Limit Definition of a Derivative



$(x, f(x)) \quad (x+h, f(x+h))$

the slope of the secant = $\dfrac{f(x+h) - f(x)}{x+h - x} = \dfrac{f(x+h) - f(x)}{h}$ → difference quotient

- A **secant line** to the graph of f must intersect it in at least two distinct points.
- A **tangent line** only need intersect the graph in one point, where the line might "just touch" the graph. (There could be other intersection points).

- We define the **slope** of the tangent line to be the (two-sided) **limit** of the difference quotient as $h \to 0$, if that limit exists.
- We denote this slope by $f'(x)$, read as "***f* prime** of x".

Definition of the Derivative

the slope of the tangent line

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

provided the limit exists

Examples Use the definition of the derivative to find the derivative of each function with respect to x.

1. $f(x) = -5x + 3$

$$f'(x) = \lim_{h \to 0} \frac{-5(x+h)+3 - (-5x+3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-5x} - 5h \cancel{+3} \cancel{+5x} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} (-5) = \boxed{-5}$$

$(x+h)(x+h) = x^2 + \underbrace{hx + hx}_{2hx} + h^2$

2. $f(x) = 2x^2 + 7x - 1$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 + 7(x+h) - 1 - (2x^2 + 7x - 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 + \cancel{7x} + 7h - \cancel{1} - \cancel{2x^2} - \cancel{7x} + \cancel{1}}{h}$$

$$= \lim_{h\to 0} (4x + 2h + 7)$$

$$= 4x + 2(0) + 7 = \boxed{4x+7}$$

3. $f(x) = \sqrt{3 - 2x}$

$$f'(x) = \lim_{h\to 0} \left(\frac{\sqrt{3-2(x+h)} - \sqrt{3-2x}}{h}\right)\left(\frac{\sqrt{3-2(x+h)} + \sqrt{3-2x}}{\sqrt{3-2(x+h)} + \sqrt{3-2x}}\right)$$

$$= \lim_{h\to 0} \frac{3 - 2(x+h) - (3-2x)}{h\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)} = \lim_{h\to 0} \frac{\cancel{3} - \cancel{2x} - 2\cancel{h} - \cancel{3} + \cancel{2x}}{\cancel{h}\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)}$$

$$= \lim_{h\to 0} \frac{-2}{\sqrt{3-2(x+h)} + \sqrt{3-2x}} = \frac{-2}{\sqrt{3-2x} + \sqrt{3-2x}} = \frac{-\cancel{2}}{\cancel{2}\sqrt{3-2x}}$$

$$= \boxed{\frac{-1}{\sqrt{3-2x}}}$$

4. $f(x) = \frac{2}{x-5}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2(x-5)}{(x+h-5)(x-5)} - \frac{2(x+h-5)}{(x-5)(x+h-5)}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{2x} - \cancel{10} - \cancel{2x} - 2h + \cancel{10}}{(x+h-5)(x-5)}}{h}$$

$$\lim_{h\to 0} \frac{-2\cancel{h}}{(x+h-5)(x-5)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-2}{(x+h-5)(x-5)}$$

$$= \frac{-2}{(x-5)(x-5)} = \boxed{\frac{-2}{(x-5)^2}}$$

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