

9/25/18

# Derivatives of Inverse Trig Functions

\* rules are based on implicit differentiation

$$y = \sin^{-1} x$$

$$\sin y = \sin(\sin^{-1} x)$$

$$\therefore \sin y = x \leftarrow \text{implicit eqn}$$

$$\text{deriv: } \cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

EX 1 Find the derivative.

A)  $y = \sin^{-1}(2x)$

$$y' = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

B)  $f(x) = -2 \cos^{-1}(3x-1) + 5x$

$$f'(x) = -2 \cdot -\frac{1}{\sqrt{1-(3x-1)^2}} \cdot 3 + 5 = \frac{6}{\sqrt{1-(3x-1)^2}} + 5$$

C)  $y = \sec^{-1}(e^{2x})$

$$y' = \frac{1}{|e^{2x}| \sqrt{(e^{2x})^2 - 1}} \cdot e^{2x} \cdot 2 = \frac{2}{\sqrt{e^{4x} - 1}}$$

EX 2 Find  $\frac{d^2y}{dx^2}$  if  $2x^3 - 3y^2 = 8$

$$\text{1st deriv: } 6x^2 - 6y \frac{dy}{dx} = 0$$

$$-6y \frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx} = \frac{-6x^2}{-6y} = \frac{x^2}{y}$$

$$\text{2nd deriv: } \frac{d^2y}{dx^2} = \frac{y \cdot 2x - x^2 \cdot 1 \frac{dy}{dx}}{y^2} = \frac{2xy - x^2 \cdot x \frac{x^2}{y}}{y^2}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

EX 3  $x^2 - xy + y^2 = 7$

Find the eqns of the tangent & normal lines to the ellipse at  $(-1, 2)$ .

$$2x - x \cdot 1 \frac{dy}{dx} + y \cdot -1 + 2y \frac{dy}{dx} = 0$$

$$-x \frac{dy}{dx} + 2y \frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx} (-x + 2y) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{-x + 2y}$$

$$T: y - 2 = \frac{4}{5}(x + 1)$$

$$N: y - 2 = -\frac{5}{4}(x + 1)$$

$$\left. \frac{dy}{dx} \right|_{(-1, 2)} = \frac{2 - 2(-1)}{-(-1) + 2(2)} = \frac{4}{5}$$