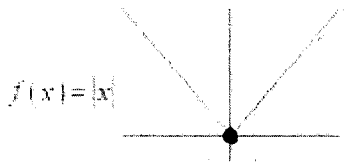
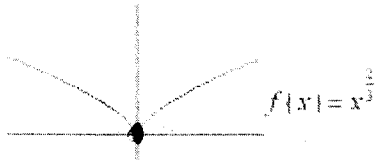
When does a derivative fail to exist???

To be differentiable, a function must be <u>continuous</u> and <u>smooth</u>.

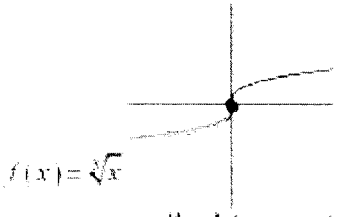
Derivatives will fail to exist at:



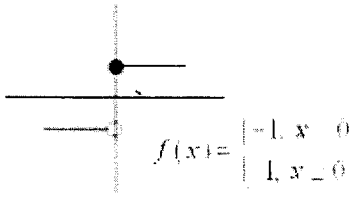
corner



cusp



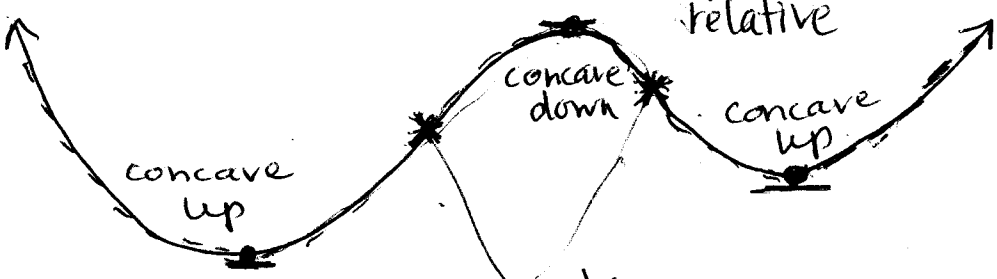
vertical tangent



discontinuity

If f is "differentiable" at x = c, then f is also continuous at x = c.

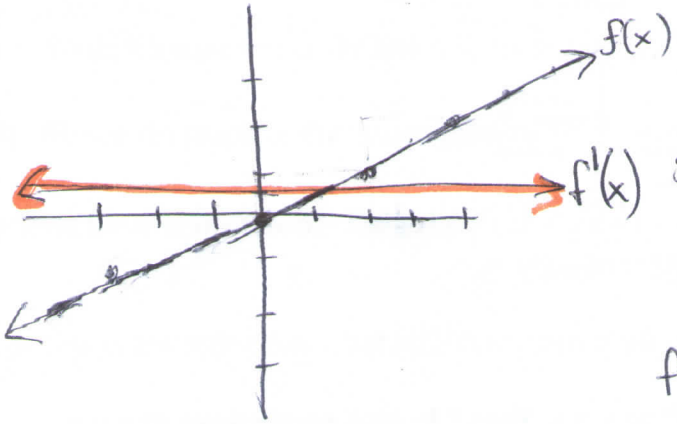
Slope of tangents to a curve	Derivative value	Original function	Graph of derivative
positive	pos.	increasing	above x-axis
negative	neg.	decreasing	below x-axis
zero horiz.	0	changing from incr/decr (where relative local max/mins)	zeros (x-intercepts)



points of inflection

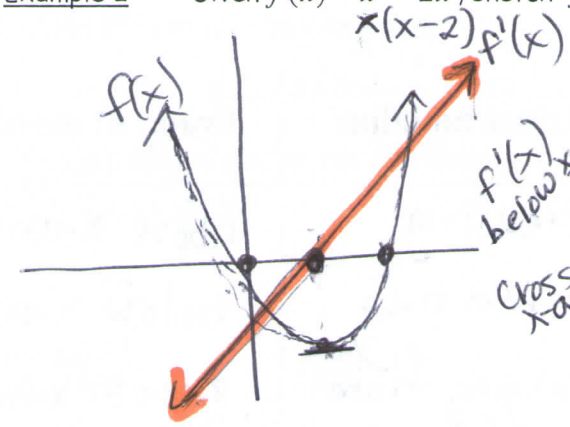
2nd Derivative value	Original function
positive	concave up
negative	concave down
zero	possible points of inflection

Example 1 Given $f(x)=\frac{1}{2}x$, sketch $f'(x)$.



slopes of tangents
pos (constant $\frac{1}{2}$)
f'(x) above x-axis

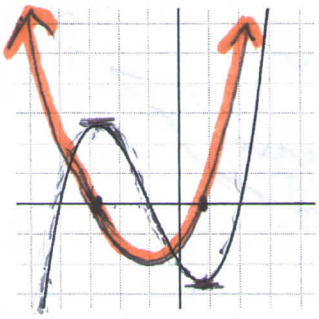
Example 2 Given $f(x)=x^2-2x$, sketch $f'(x)$.



slopes of tang.
f'(x) below x-axis neg until x=1
cross x-axis ← zero at x=1
pos until ∞
(f'(x) above x-axis

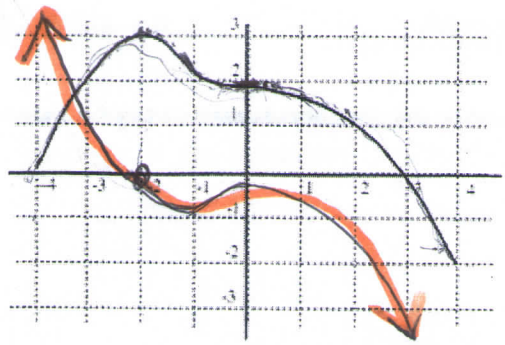
Example 3 Given the graph of $f(x)$ below, sketch $f'(x)$.

A.



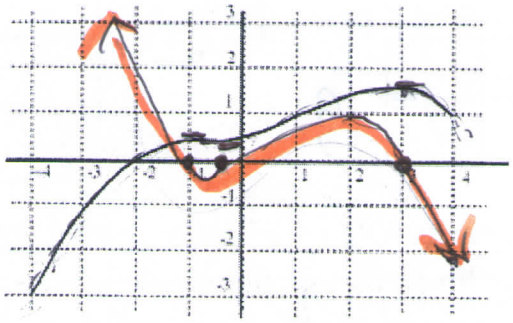
slopes of tang.
pos until x = -2.75
neg until x = .75
} zero
pos until ∞

B.



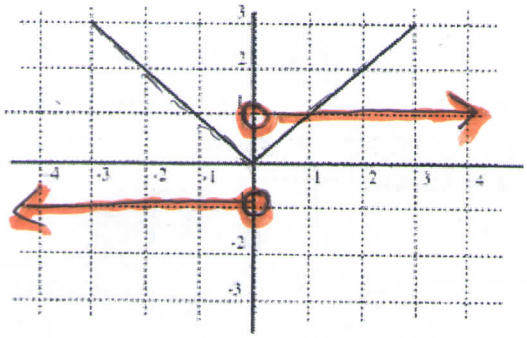
pos until $x=-2$
neg until kinda $x=0$
neg until ∞

C.



pos. until $x=-1$.
neg until $x=-.5$
pos until $x=3$
neg until ∞.

D.



not smooth
no deriv. at $x=0$

neg (constant -1) until $x=0$
pos (constant 1) ∞