Error Worksheet

1.

n	$\lvert a_n \rvert$
1	$\lvert 1 \rvert = 1$
2	$\lvert -\frac{1}{16} \rvert = .0625$
3	$\lvert \frac{1}{81} \rvert = .0123$
4	$\lvert -\frac{1}{256} \rvert = .003$
5	$\lvert \frac{1}{625} \rvert = .0016 \leftarrow .001$
6	$\lvert -\frac{1}{1296} \rvert = .000772$

$\boxed{5 \text{ terms}}$

2. a. $\sum (-1)^{n+1} \cdot \frac{1}{n^p} = \sum (-1)^{n+1} \left(\frac{1}{n}\right)^p$ converges if p is positive $\boxed{p>0}$

b. $k+1$ term would be $(-1)^{k+1+1} \cdot \frac{1}{(k+1)^p}$

$$\left| (-1)^{k+2} \cdot \frac{1}{(k+1)^p} \right| = \boxed{\frac{1}{(k+1)^p}}$$

3. a) $\sum_{n=1}^{10} (-1)^{n+1} \cdot \frac{1}{n^n} = \frac{1}{1^1} - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} - \frac{1}{6^6} + \frac{1}{7^7} - \frac{1}{8^8} + \frac{1}{9^9} - \frac{1}{10^{10}} = \boxed{.78343}$

b) $\lvert 11^{th} \text{ term} \rvert = \left| (-1)^{12} \cdot \frac{1}{11^{11}} \right| = \boxed{3.505 \times 10^{-12}}$

c) $\boxed{\text{decreased}}$ ... more terms $\Rightarrow$ better approximation $\Rightarrow$ less error

4. a. $\lvert 5^{th} \text{ term} \rvert = \boxed{\frac{1}{256}}$

b. geometric series sum $= \frac{1}{1-(-.25)} = \boxed{\frac{4}{5}}$ $\quad 1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} = .796875$

exact − approx. $= \frac{4}{5} - \frac{1}{256} = \boxed{.003125}$ (less than e.b.)

5. $\left|11^{th} \text{ term}\right| = \left|(-1)^{11} \cdot \frac{1}{11}\right| = \frac{1}{11} = \boxed{.09}$

6.

n	$\lvert a_n \rvert$
1	1
2	.125
3	.037
4	
$\vdots$	
9	.00137
10	.001
11	.0007513

$\boxed{n=11}$

i.e. if you use 10 terms for the approximation, the e.b. will be $|11^{th} \text{ term}|$

7. $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \underbrace{x - \frac{x^3}{3!}} + \frac{x^5}{5!} - \dots$

error bound $= \left|\frac{x^5}{5!}\right| = \left|\frac{.5^5}{5!}\right| = \boxed{.0002604}$

8. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

build $e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum \frac{(-1)^n x^n}{n!}$

for $x = 2$, $p(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$

error bound $= |6^{th} \text{ term}| = \left|\frac{-x^5}{5!}\right| = \left|-\frac{2^5}{5!}\right| = \boxed{\frac{2^5}{5!}}$

9. $f(x) = \ln x$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2}$

$f'''(x) = 2x^{-3}$

$f^{(4)}(x) = -6x^{-4}$

$a = 1 \quad x = 1.2$

$|f^{(4)}(1)| = |-6| = 6 \leftarrow$ bigger

$|f^{(4)}(1.2)| = \left|-\frac{625}{216}\right| = 2.89352$

error bound $= \left|\frac{|6|(1.2-1)^4}{4!}\right| = .0004$ $\boxed{B}$

10. $\frac{1}{\sqrt{e}} = e^{-\frac{1}{2}} = .6065306597$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots$$

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$P_3\left(-\frac{1}{2}\right) = .6041666667$$

$f(x) = p(x) + \text{remainder}$

remainder = $\underset{f(x)}{\text{actual}}$ − $\underset{p(x)}{\text{approximation}}$

$.6065306597 - .6041666667 = .002363993$

$\boxed{A}$