Precalculus Name: ______________________

Notes --- (3.3 & 3.4) Logarithmic Functions

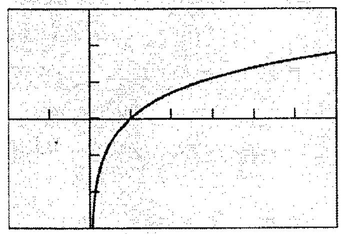
➢ Logarithmic functions are inverses of exponential functions.

$y = \log_b x$

inverse: $x = \log_b y$

$b^x = y$

BASIC FUNCTION The Natural Logarithmic Function



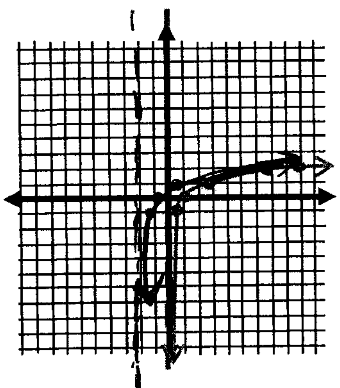
[–2, 6] by [–3, 3]

$f(x) = \ln x$
Domain: $(0, \infty)$
Range: All reals
Continuous on $(0, \infty)$
Increasing on $(0, \infty)$
No symmetry
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptote: $x = 0$
End behavior: $\lim_{x \to \infty} \ln x = \infty$

Ex1) Describe how to transform the graph of $y = \ln x$ **or** $y = \log x$ into the graph of the given function. Then sketch the given function.

(a) $g(x) = \ln(x + 2)$

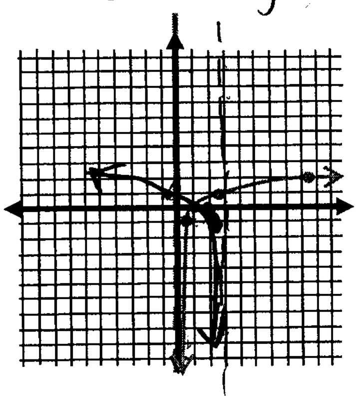
shift left 2



(b) $h(x) = \ln(3 - x)$

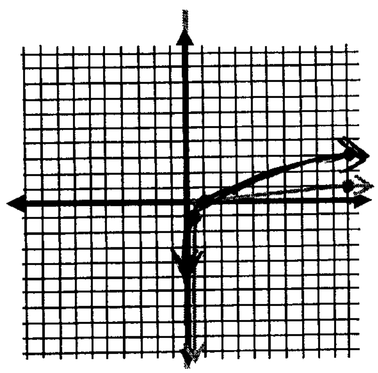
$\ln(-(x-3))$
$\ln(-x+3)$

shift right 3
refl. over y-axis



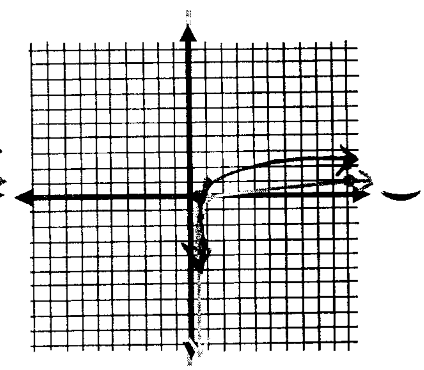
(c) $g(x) = 3 \log x$

vert. stretch *3



(d) $h(x) = 1 + \log x$

[illegible] 1



CHANGING BETWEEN EXPONENTIAL & LOGARITHMIC FORM

If $x > 0$, $b > 0$, & $b \neq 1$, then $y = \log_b x$ if and only if $x = b^y$

Ex2) Write each of the following in logarithmic or exponential form:

Log Form		**Exp Form**
a) $\log_2 8 = 3$	→	$2^3 = 8$
b) $\log_{27} 3 = \frac{1}{3}$	→	$27^{1/3} = 3$
c) $\log_{1/2} 16 = -4$	→	$\left(\frac{1}{2}\right)^{-4} = 16$
d) $\log_{25} 125 = \frac{3}{2}$	→	$25^{3/2} = 125$

Exp Form		**Log Form**
e) $5^2 = 25$	→	$\log_5 25$
f) $9^{1/2} = 3$	→	$\log_9 3$
g) $(1/4)^{-3} = 64$	→	$\log_{1/4} 64$
h) $64^{-1/6} = 1/2$	→	$\log_{64} \frac{1}{2}$

- Logarithms with base 10 are called common logs & are written without a base.
- Logarithms with base e are called natural logs & are written with "LN" instead of log $\log_e x = \ln x$

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Ex3) Evaluate each of the following logs:

(a) $\log_5 125 = 3$ $\quad 5^x = 125$

(b) $\log_7 1 = 0$ $\quad 7^x = 1$

(c) $\log_9 9^4 = 4$

(d) $11^{\log_{11} 7} = 7$

(e) $\log_8 32 = \frac{5}{3}$ $\quad 8^x = 32 \quad (2^3)$

(f) $\log_4 \frac{1}{64} = -3$ $\quad 4^x = \frac{1}{64}$

(g) $\log_3 \frac{1}{9} = -2$ $\quad 3^x = \frac{1}{9}$

(h) $\log_{1/25} 125 =$ $\quad \frac{1}{25}^x = \quad x = 3 \quad x = -\frac{3}{2}$

When in this form $\log_b x$ ASK YOURSELF **"b to what power equals x"**

Ex4) Evaluate each of the following:

(a) $\log 100 = 2$ $\quad 10^x = 100$

(b) $\log \sqrt[5]{10} = \frac{1}{5}$ $\quad 10^x = \sqrt[5]{10}$

(c) $\log \frac{1}{1000} = -3$ $\quad 10^x = \frac{1}{1000}$

(d) $10^{\log 6} = 6$

Ex5) Solve the simple logarithmic equations below by changing them to exponential form:

(a) $\log x = 3$ $\quad 10^3 = x$ $\quad \boxed{1000 = x}$

(b) $\log_2 x = 5$ $\quad 2^5 = x$ $\quad \boxed{32 = x}$

Ex6) Evaluate each of the following:

(a) $\ln \sqrt{e} = \frac{1}{2}$ $\quad e^x = \sqrt{e}$

(b) $\ln e^5 = 5$ $\quad e^x = e^5$

(c) $e^{\ln 4} = 4$

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$

- **Product rule:** $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule:** $\log_b \frac{R}{S} = \log_b R - \log_b S$
- **Power rule:** $\log_b R^c = c \log_b R$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Ex7) Expand each of the following:

(a) $\log (8xy^4)$ **(b)** $\ln \left(\frac{\sqrt{x^2+5}}{x} \right)$

(a) $\log 8 + \log x + \log y^4 = \log 8 + \log x + 4 \log y$

(b) $\ln \sqrt{x^2+5} - \ln x = \ln (x^2+5)^{1/2} - \ln x = \frac{1}{2} \ln(x^2+5) - \ln x$

Ex8) Condense the following logarithmic expression: $\ln x^5 - 2 \ln (xy) = \ln x^5 - \ln (xy)^2 = \ln \frac{x^5}{x^2 y^2} = \ln \frac{x^3}{y^2}$

Ex9) Given that $\ln 5 = a$ & $\ln 7 = b$ determine each of the following:

a) $\ln 35 = a+b$ b) $\ln (5/7) = a-b$ c) $\ln 175 = 2a+b$ d) $\log_5 7 = \frac{b}{a}$ e) $\log_7 35 =$ f) $\log_5 175 = \frac{2a+b}{a}$

a) $\ln(5 \cdot 7)$
$\ln 5 + \ln 7$
$a + b$

b) $\ln 5 - \ln 7$
$a - b$

c) $\ln (5^2 \cdot 7)$
$\ln 5^2 + \ln 7$
$2 \ln 5 + \ln 7$
$2a + b$

d) $\frac{\ln 7}{\ln 5}$ $\quad \frac{\log_e 7}{\log_e 5}$ change-of-base formula
$\frac{b}{a}$

e) $\frac{\ln 35}{\ln 7}$

f) $\frac{\ln 175}{\ln 5}$ see part c
$\frac{2a+b}{a}$