Precalculus **Name:** ______________________

3.1------Exponential & Logistic Functions

DEFINITION-------Exponential Functions

An **EXPONENTIAL FUNCTION** is a function that can be written in the form

$f(x) = ab^x$
- a is non-zero number
- b is a positive number
- $b \neq 1$

The constant a is called the initial value of f

(Notice this is the value of f at $x = 0$)

The constant b is the base

(Notice this is the ONLY number being raised to the x power…a is NOT)

************************IDENTIFYING EXPONENTIAL FUNCTIONS************************

Ex1) For each of the following state whether the function is exponential & its initial value, base, and exponent.

(a) $f(x) = 3^x$

yes

$a = 1$

$b = 3$

exp. = x

(b) $g(x) = 6x^{-4} = \frac{6}{x^4}$

no

(c) $h(x) = -2 \cdot 1.5^x$

yes

$a = -2$

$b = 1.5$

exp = x

(d) $k(x) = 7 \cdot 2^{-x}$

yes $= 7 \cdot (2^{-1})^x$

$= 7\left(\frac{1}{2}\right)^x$

$a = 7$

$b = \frac{1}{2}$

exp = x

(e) $q(x) = 5 \cdot 6^{\pi}$

no (constant)

************************EVALUATING EXPONENTIAL FUNCTIONS************************

Ex2) Evaluate each of the following for $f(x) = 2^x$:

(a) $f(4) = 16$

2^4

(b) $f(0) = 1$

2^0

(c) $f(-3) = \frac{1}{8}$

$2^{-3} = \frac{1}{2^3}$

(d) $f(½) = \sqrt{2}$

$2^{\frac{1}{2}} = \sqrt{2}$

(e) $f(-3/2) =$

$2^{-3/2}$

$\frac{1}{(\sqrt{2})^3}$ or $\frac{1}{\sqrt{2^3}}$

$\frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

***************************FINDING EXPONENTIAL FUNCTIONS***************************

Ex3) Given its table of values or its graph, find the equation of the exponential function:

(a)

x	$f(x)$
-2	6/25
-1	6/5
0	6
1	30
2	150

$* 5$

"a" value

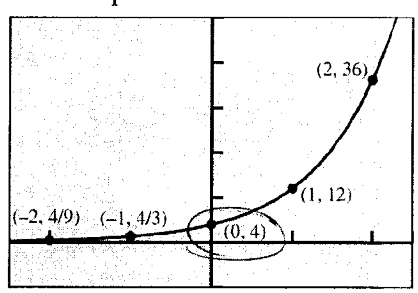
$a = 6$

$b = 5$

$f(x) = 6 \cdot 5^x$

$b > 1$
growth

(b)



[−2.5, 2.5] by [−10, 50]

$a = 4$

$b = 3$

$f(x) = 4 \cdot 3^x$

NOW YOU TRY ☺

(c)

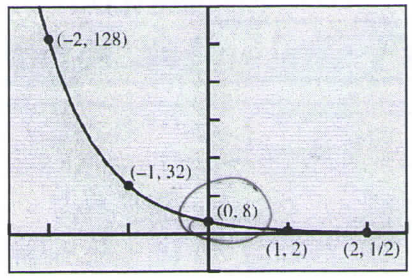
x	$f(x)$
-2	56
-1	28
0	14
1	7
2	7/2

$a = 14$
$b = \frac{1}{2}$

$f(x) = 14 \cdot \left(\frac{1}{2}\right)^x$

$0 < b < 1$
decay

(d)



[−2.5, 2.5] by [−25, 150]

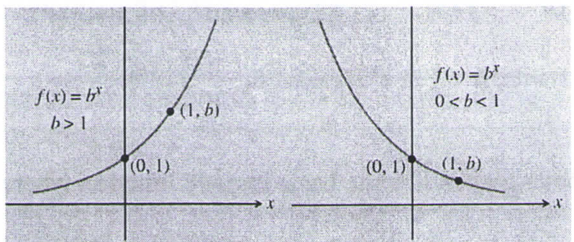
$a = 8$
$b = \frac{1}{4}$

$f(x) = 8 \cdot \left(\frac{1}{4}\right)^x$

******************************Transforming Exponential Functions******************************

Exponential Functions $f(x) = b^x$

Domain: All reals
Range: $(0, \infty)$
Continuous
No symmetry: neither even nor odd
Bounded below, but not above
No local extrema
Horizontal asymptote: $y = 0$
No vertical asymptotes



Graphs of $f(x) = b^x$ for (a) $b > 1$ and (b) $0 < b < 1$.

end behavior

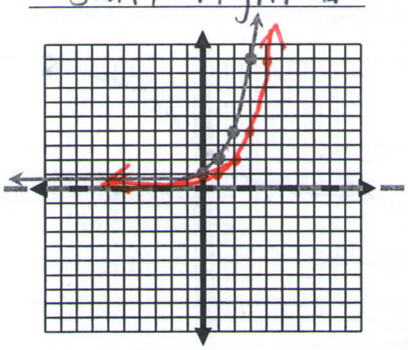
If $b > 1$ (see Figure 3.3a), then
- f is an increasing function,
- $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = \infty$.

If $0 < b < 1$ (see Figure 3.3b), then
- f is a decreasing function,
- $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 0$.

Ex4) Describe how to transform the graph of $f(x) = 2^x$ into each of the given functions and sketch.

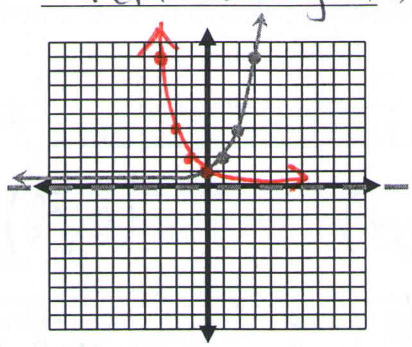
$g(x) = 2^{x-1}$

shift right 1



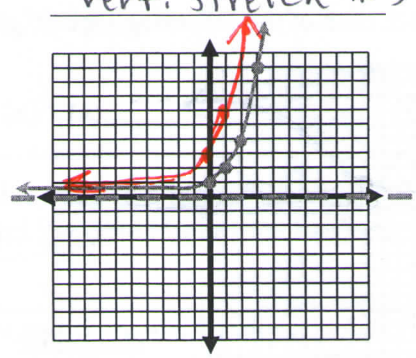
$h(x) = 2^{-x}$

refl. over y-axis



$k(x) = 3 \cdot 2^x$

vert. stretch * 3



***************************************THE NATURAL BASE e ***************************************

DEFINITION The Natural Base e

$$e = \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x$$

$e \approx$ 2.718281828459...
(irrational)

Ex5) Describe how to transform $f(x) = e^x$ into each of the following functions:

$g(x) = e^{2x} + 2$

horiz. shrink * $\frac{1}{2}$
shift up 2

$h(x) = -e^{x-3}$

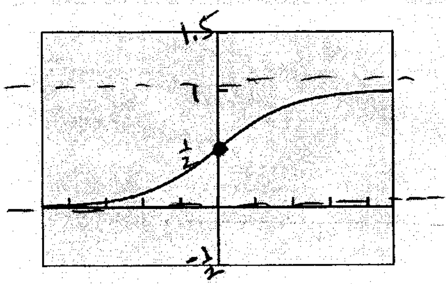
right 3
refl. over x-axis

$k(x) = \frac{1}{2} e^{-x}$

refl. over y-axis
vert. shrink * $\frac{1}{2}$

**********************************LOGISTIC FUNCTIONS**********************************

BASIC FUNCTION The Logistic Function



[–4.7, 4.7] by [–0.5, 1.5]

FIGURE 3.8 The graph of $f(x) = 1/(1 + e^{-x})$.

$f(x) = \frac{1}{1 + e^{-x}}$

Domain: All reals
Range: (0, 1)
Continuous
Increasing for all x
Symmetric about (0, 1/2), but neither even nor odd
Bounded below and above
No local extrema
Horizontal asymptotes: $y = 0$ and $y = 1$
No vertical asymptotes
End behavior: $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 1$

DEFINITION-----------Logistic Growth Functions
A LOGISTIC GROWTH FUNCTION in x is a function that can be written in the form

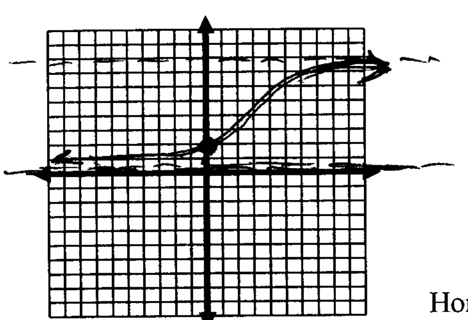
$$f(x) = \frac{c}{1 + a \cdot b^x} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where ***a***, ***b***, ***c***, & ***k*** are positive constants, ***b*** < 1 & ***c*** is called the limit to growth.

- All logistic growth functions have graphs like the basic logistic function where the end behavior can be described as: $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = c$
- All logistic growth functions are bounded by asymptotes $y =$ 0 & $y =$ c
- All logistic growth functions have a range (0, c)

*******************************GRAPHING LOGISTIC FUNCTIONS*******************************

Ex6) Sketch each of the following logistic growth functions, identify the y-int & horizontal asymptotes. block = 2units



(a) $f(x) = \frac{8}{1 + 3 \bullet 0.7^x}$

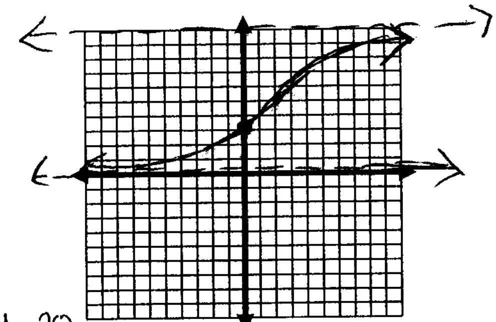
y-int: (0, 2)

Horizontal Asymptotes: $y=0$, $y=8$

(b) $g(x) = \frac{20}{1 + 2e^{-3x}}$

y-int: $(0, \frac{20}{3})$

Horizontal Asymptotes: $y=0$, $y=20$



$f(0) = \frac{8}{1+3 \bullet 0.7^0} = \frac{8}{1+3} = 2$

$g(0) = \frac{20}{1+2e^{-3(0)}} = \frac{20}{1+2} = \frac{20}{3}$