

NOTES 3.5: Solving Exponential & Logarithmic Equations

ONE-TO-ONE PROPERTIES:

For any exponential function $f(x) = b^x$
 If $b^u = b^v$, then $u = v$

Ex) $7^4 = 7^x$, then $x = 4$

For any logarithmic function $f(x) = \log_b x$
 If $\log_b v = \log_b u$, then $u = v$

Ex) $\log 12 = \log(x)$, then $x = 12$

Makes sense right? Be sure to notice though, that the **BASES** must **MATCH** to use this property.

Ex1) Solve: $4^{3x} = 8^{x+1}$

$$(2^2)^{3x} = (2^3)^{x+1}$$

$$2^{6x} = 2^{3x+3}$$

$$6x = 3x + 3$$

$$3x = 3$$

$$\boxed{x = 1}$$

Ex2) Solve: $20\left(\frac{1}{2}\right)^{x/3} = 5$

$$\left(\frac{1}{2}\right)^{x/3} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{x/3} = \left(\frac{1}{2}\right)^2$$

$$\frac{x}{3} = 2$$

$$\boxed{x = 6}$$

When it is not "convenient" to write each side with the same base, you can simply take the log of each side. Then use the power property of logs to bring the exponent down to solve.

Ex3) $3^{x-2} = 7$

$$\log 3^{x-2} = \log 7$$

$$\frac{(x-2) \cdot \log 3}{\log 3} = \frac{\log 7}{\log 3}$$

$$x-2 = \log_3 7$$

$$\boxed{x = 2 + \log_3 7}$$

Solving logarithmic equations:

Ex6) $\log_5(3x+1) = 2$

$$5^2 = 3x+1$$

$$25 = 3x+1$$

$$24 = 3x$$

$$\boxed{8 = x}$$

Ex4) $10^{2x-3} + 4 = 21$

$$10^{2x-3} = 17$$

$$\log 10^{2x-3} = \log 17$$

$$\frac{(2x-3) \cdot \log 10}{\log 10} = \frac{\log 17}{\log 10}$$

$$2x-3 = \log_{10} 17$$

$$2x = 3 + \log_{10} 17$$

(for this one round to the nearest hundredth)

Ex5) $9^{x+1} = 11^{x-3}$

$$\log 9^{x+1} = \log 11^{x-3}$$

$$(x+1) \cdot \log 9 = (x-3) \log 11$$

$$x \log 9 + \log 9 = x \log 11 - 3 \log 11$$

$$x \log 9 - x \log 11 = -3 \log 11 - \log 9$$

$$x(\log 9 - \log 11) = -3 \log 11 - \log 9$$

$$\boxed{x = \frac{3 + \log_{10} 17}{2}}$$

$$\boxed{x = \frac{-3 \log 11 - \log 9}{\log 9 - \log 11}}$$

$$\boxed{46.80}$$

Ex7) $\log x^2 = 2$

$$10^2 = x^2$$

$$100 = x^2$$

$$x = \pm \sqrt{100} = \boxed{\pm 10}$$

You should ALWAYS check your answers when solving equations, but this becomes even more important when dealing with log equations since they have restricted domains.

Ex8) $\log(5x) + \log(x-1) = 2$

$$\log[5x(x-1)] = 2$$

$$10^2 = 5x^2 - 5x$$

$$0 = 5x^2 - 5x - 100$$

$$0 = 5(x^2 - x - 20)$$

$$0 = 5(x-5)(x+4)$$

$$x-5=0 \quad x+4=0$$

$$\boxed{x=5} \quad x = -4 \text{ extraneous}$$

Now You Try ☺

10) $\log_4(3x-8) = 3$

$$4^3 = 3x-8$$

$$64 = 3x-8$$

$$72 = 3x$$

$$\boxed{24 = x}$$

12) $\log_{27} m = 4/3$

14) $\log_2 x + \log_2(x^2 - 9) = \log_2 16x$

16) $3^{x+3} = 2^x$
 $\log 3^{x+3} = \log 2^x$

$$(x+3) \log 3 = x \log 2$$

$$x \log 3 + 3 \log 3 = x \log 2$$

$$3 \log 3 = x \log 2 - x \log 3$$

$$\boxed{x = -8.13}$$

18) $\log x + \log(x-3) = 1$

Ex9) $\ln(3x-2) + \ln(x-1) = 2 \ln x$

$$\ln[(3x-2)(x-1)] = \ln x^2$$

$$3x^2 - 3x - 2x + 2 = x^2$$

$$2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$$2x-1=0 \quad x-2=0$$

$$2x=1 \quad \boxed{x=2}$$

$$x = \frac{1}{2}$$

11) $\ln(5x-1) = \ln(x+2) + \ln 2$

$$\ln(5x-1) = \ln(2x+4)$$

$$5x-1 = 2x+4$$

$$3x=5 \quad \boxed{x=5/3}$$

13) $e^{2x} - 28 = 3e^x$

15) $36^{x+2} = 6^{x-1}$

$$(6^2)^{x+2} = 6^{x-1}$$

$$6^{2x+4} = 6^{x-1}$$

$$2x+4 = x-1$$

$$\boxed{x = -5}$$

$$\log(4x-1) = \log(x+1) + \log 2$$

17) $3 \log 3 = x(\log 2 - \log 3)$

$$x = \frac{3 \log 3}{\log 2 - \log 3}$$

19) $e^{2x} - 5e^x + 6 = 0$