Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then $P(t) = P_0(1+r)^t$ where P_0 is the initial population, r is expressed as a decimal, and t is time in years.

- r is positive: P is an exponential growth function & it's growth factor is $1+r$
- r is negative: P is an exponential decay function & it's decay factor is $1+r$

Ex 1) Tell whether the population model is an exponential growth function or exponential decay function, and find the constant percentage rate of growth or decay:

(a) Population of San Jose: $P(t) = 782{,}248 \bullet 1.0135^t$

$1+r = 1.0135$
$r = .0135$
1.35% growth

(b) Population of Detroit: $P(t) = 1{,}203{,}368 \bullet 0.9858^t$

$1+r = 0.9858$
$r = -.0142$
−1.42% decay

Ex 2) Determine the exponential function with initial value = 12, increasing at a rate of 8% per year.

.08

$p(t) = 12(1+.08)^t$

$p(t) = 12(1.08)^t$

Ex 3) Suppose a culture of 100 bacteria is put into a Petri dish and the culture doubles every hour. Determine a function B(t) that will model the situation and then, using your calculator's "intersect" function, predict when the number of bacteria will be 350,000.

P_0

100%
1.00

$B(t) = 100(1+1)^t = 100(2)^t$

$\underset{y_1}{350000} = \underset{y_2}{100(2)^t}$ intersection at (11.77, 350000) $\boxed{11.77 \text{ hrs}}$

The number of atoms of a specific element that change from a radioactive state to a nonradioactive state is a fixed fraction per unit time. The process is called ***radioactive decay, and the time it takes for half of a sample to change its state is the ***half-life*** of the radioactive substance.***

Ex 4) Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially. Determine a model $f(t)$ for the situation. Then, using the intersect function in your calculator, find the time t when there will be 1 gram of the substance remaining.

P_0

$r = -50\% = -.5$

$f(t) = 5(1-.5)^{t/20} = 5\left(\frac{1}{2}\right)^{t/20}$

$1 = 5\left(\frac{1}{2}\right)^{t/20}$

intersection at (46.44, 1) $\boxed{46.44 \text{ day}}$

Ex 5) Scientists have established that atmospheric pressure at sea level is 14.7 lbs per sq. in, and the pressure is reduced by half for each 3.6 miles above sea level. This rule holds for altitudes up to 50 mi above sea level. Determine a function $P(h)$ that models the situation. Then, using the intersect feature on your calculator, find the altitude above sea level at which the atmospheric pressure is 4 pounds per square inch.

$r = -.5$ $\qquad P_0$

$$P(h) = 14.7\left(\tfrac{1}{2}\right)^{\frac{h}{3.6}}$$

$$4 = 14.7\left(\tfrac{1}{2}\right)^{\frac{h}{3.6}}$$

$\boxed{6.76 \text{ miles}}$

Ex 6) Use the data in the table below to determine the exponential regression model representing the U.S. population with respect to time. Then, use this model ~~the intersect function on your calculator~~ to predict the population in 2000 & compare the result with the listed value for 2000. ~~What would be a good window for this?~~

US POPULATION (IN MILLIONS)											
	0	10	20								
Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Pop.	76.2	92.2	106.0	123.2	132.2	151.3	179.3	203.3	226.5	248.7	281.4

$x = 100$

$$y = 80.5574 \bullet (1.012887)^x$$

100 $\qquad \boxed{289.8}$ millions

Exponential growth is NOT restricted. However, in many populations the growth begins exponentially but eventually slows and approaches a <u>limit</u> to <u>growth</u> also called the ***maximum sustainable population or the ***carrying capacity***. This limited growth suggests a logistic model.

Ex 7) Use the data in the table below to create a logistic regression model for the situation. Then, predict the maximum sustainable population for Florida and Pennsylvania.

POPULATIONS OF TWO US STATES (IN MILLIONS)											
	0	10	20								
Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Florida	0.5	0.8	1.0	1.5	1.9	2.8	5.0	6.8	9.7	12.9	16.0
Pennsylvania	6.3	7.7	8.7	9.6	9.9	10.5	11.3	11.8	11.9	11.9	12.3

$$F(x) = \frac{28.02}{1 + 81.90e^{-.05x}} \qquad P(x) = \frac{12.58}{1 + .94e^{-.03x}}$$

m.s.p. = 28.02 million $\qquad$ m.s.p. = 12.58 million

Ex 8) HSHS has 1200 students. Gabe, Tim, and Ashley start a rumor about Ms. Powell, which spreads logistically so that $S(t) = \dfrac{1200}{1 + 39 \bullet e^{-0.9t}}$ models the number of students who have heard the rumor by the end of t days, where t = 0 is the day the rumor begins to spread.

$t = 0$ $\qquad \dfrac{1200}{1 + 39 \cdot e^{-.9(0)}} = \dfrac{12}{4}$

(a) How many students have heard the rumor by the end of day 0?

(b) How long does it take for 1000 students to hear the rumor?

$$1000 = \frac{1200}{1 + 39e^{-.9t}}$$

$\boxed{t = 5.86 \text{ days}}$

Writing Exponential and Logistic Functions

Ex 9) Determine the exponential function that satisfies the given conditions:
Initial value = 7, decreasing at a rate of 2.3% each year

P_0 $\quad r = -.023$

$$p(t) = 7(1 + -.023)^t = 7(.977)^t$$

Ex 10) Find a logistic function of the form: $f(x) = \frac{c}{1+a\cdot b^x}$ that satisfies the following conditions:

a) Initial value = 10 $(0, 10)$
Limit to growth = 30 $c = 30$
Passing through (1, 30/7)

$$10 = \frac{30}{1+a\cdot b^0}$$

$$\frac{10}{1} = \frac{30}{1+a}$$

$$10(1+a) = 30$$

$$1+a = 3$$

$$a = 2$$

$$f(x) = \frac{30}{1+2\cdot b^x}$$

$$\frac{30}{7} = \frac{30}{1+2\cdot b^1}$$

$$30(1+2b) = 210$$

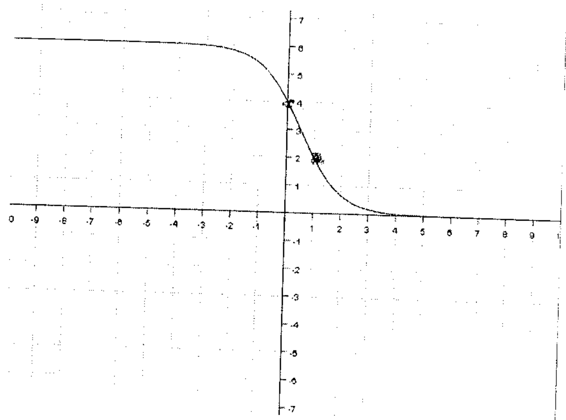
$$1+2b = 7$$

$$2b = 6$$

$$b = 3$$

$$\boxed{f(x) = \frac{30}{1+2\cdot 3^x}}$$

b)



$(0, 4)$ $\quad c = 6$

$$4 = \frac{6}{1+a\cdot b^0}$$

$$\frac{4}{1} = \frac{6}{1+a}$$

$$4(1+a) = 6$$

$$1+a = \frac{3}{2}$$

$$a = \frac{1}{2}$$

$$f(x) = \frac{6}{1+\frac{1}{2}\cdot b^x}$$

$(1, 2)$

$$2 = \frac{6}{1+\frac{1}{2}\cdot b^1}$$

$1+\frac{1}{2}b$

$\frac{1}{2}b$

b

$f(x) = \frac{6}{1+}$

$2(1+\frac{1}{2}b) = 6$

$2($